7.5 Trapezoids and Kites

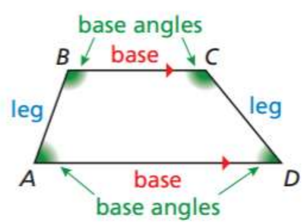
Monday, January 23, 2017 2:59 PM

What You Will Learn

- Use properties of trapezoids.
- Use the Trapezoid Midsegment Theorem to find distances.
- Use properties of kites.
- Identify quadrilaterals.

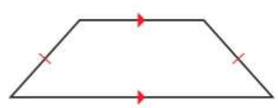
Using Properties of Trapezoids

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are the **bases**.

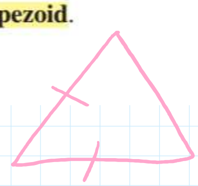


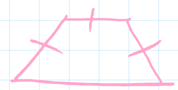
Base angles of a trapezoid are two consecutive angles whose common side is a base. A trapezoid has two pairs of base angles. For example, in trapezoid $ABCD$, $\angle A$ and $\angle D$ are one pair of base angles, and $\angle B$ and $\angle C$ are the second pair. The nonparallel sides are the **legs** of the trapezoid.

If the legs of a trapezoid are congruent, then the trapezoid is an **isosceles trapezoid**.

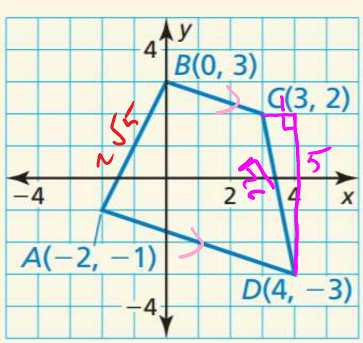


Isosceles trapezoid





Show that *ABCD* is a trapezoid and decide whether it is isosceles.



$m\,\overline{BC} = \frac{-1}{3}$

$m\,\overline{AD} = \frac{-2}{6} = \frac{-1}{3}$

$m\,\overline{AB} = \frac{4}{2} = 2 \approx +$

$m\,\overline{CD} = \frac{-5}{1} = -5 \approx -$

Proves Trapezoid

Not Isoscelis

$D = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$a^2+b^2=c^2$

$1^2+5^2=c^2$

$1+25=c^2$

$\sqrt{26}=c$

26

2 13

$AB = \sqrt{(-2-0)^2+(-1-3)^2}$

$\sqrt{(-2)^2+(-4)^2}$

$\sqrt{4+16}$

$\sqrt{20}$

$2\sqrt{5}$

20

2 10

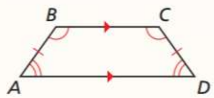
2 5

Theorem 7.14 Isosceles Trapezoid Base Angles Theorem

If a trapezoid is isosceles, then each pair of base angles is congruent.

If trapezoid $ABCD$ is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.

Proof Ex. 39, p. 405

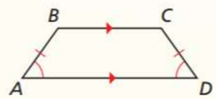


Theorem 7.15 Isosceles Trapezoid Base Angles Converse

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid $ABCD$ is isosceles.

Proof Ex. 40, p. 405

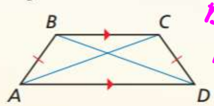


Theorem 7.16 Isosceles Trapezoid Diagonals Theorem

A trapezoid is isosceles if and only if its diagonals are congruent.

Trapezoid $ABCD$ is isosceles if and only if $\overline{AC} \cong \overline{BD}$.

Proof Ex. 51, p. 406



$BD = AC$

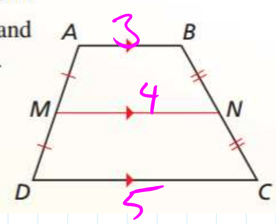
$\overline{BD} \cong \overline{AC}$

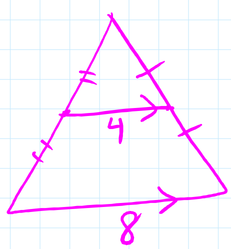
Theorem 7.17 Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base, and its length is one-half the sum of the lengths of the bases.

If $\overline{MN}$ is the midsegment of trapezoid $ABCD$, then $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, and $MN = \frac{1}{2}(AB + CD)$.

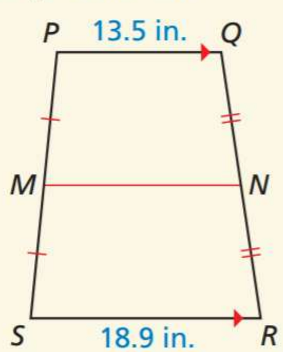
Proof Ex. 49, p. 406





$$\frac{5+3}{2} = 4$$

In the diagram, $\overline{MN}$ is the midsegment of trapezoid $PQRS$. Find MN.

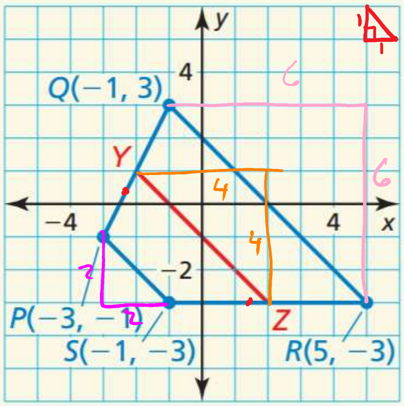


$\boxed{MN = 16.2 \text{ in}}$

$$\frac{13.5 + 18.9}{2}$$

$$\frac{32.4}{2} = 16.2$$

Find the length of midsegment $\overline{YZ}$ in trapezoid $PQRS$.



$\boxed{YZ = 4\sqrt{2}}$

$2^2+2^2=c^2$
$4+4=c^2$
$\sqrt{8}=c$
$2\sqrt{2}$

$6^2+6^2=c^2$
$36+36=c^2$
$72=c^2$
$\sqrt{72}=c$
$2\cdot3\sqrt{2}$
$6\sqrt{2}$

$4^2+4^2=c^2$
$16+16=c^2$
$\sqrt{32}=c$
$2\cdot2\sqrt{2}$
$4\sqrt{2}$

$\frac{2\sqrt{2}+6\sqrt{2}}{2}$

$\frac{8\sqrt{2}}{2}=4\sqrt{2}$

$2x+6x$
$8x$

$1^2+1^2=c^2$
$1+1=c^2$
$\sqrt{2}=c\approx1.414$

Using Properties of Kites

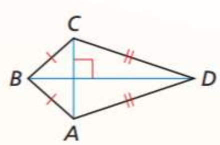
A **kite** is a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.

Theorem 7.18 Kite Diagonals Theorem

If a quadrilateral is a kite, then its diagonals are perpendicular.

If quadrilateral $ABCD$ is a kite, then $\overline{AC} \perp \overline{BD}$.

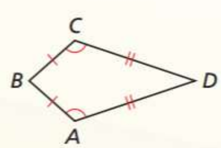
Proof p. 401



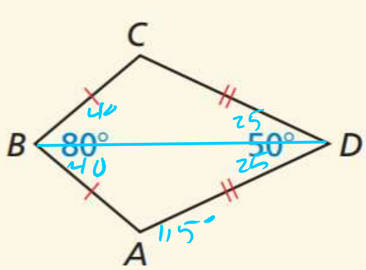
Theorem 7.19 Kite Opposite Angles Theorem

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral $ABCD$ is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \not\cong \angle D$.



Find $m\angle C$ in the kite shown.



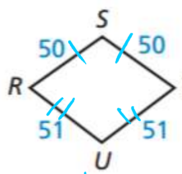
$m\angle C = 115°$

$(n-2)180$
$(4-2)180=360$

$80+50+2\alpha=360$
$130+2\alpha=360$
$-130 \quad -130$
$\frac{2\alpha}{2}=\frac{230}{2}$
$\alpha=115$

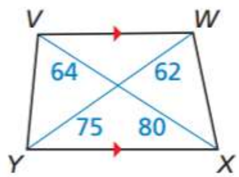
Give the most specific name for the quadrilateral. Explain your reasoning.

8.



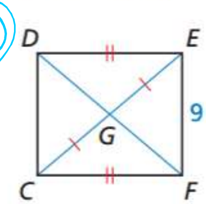
Kite

9.



Trapezoid

10.



Quadrilateral

Quadrilateral Parallelogram Rhombus Rectangle Square
Trapizoid Isoceles Trapizoid Kite

Practice sec 7.5 **pg. 403: 1-17EO, 21-33EO**