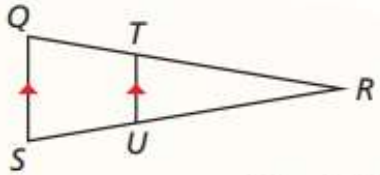
What You Will Learn

- Use the Triangle Proportionality Theorem and its converse.
- Use other proportionality theorems.

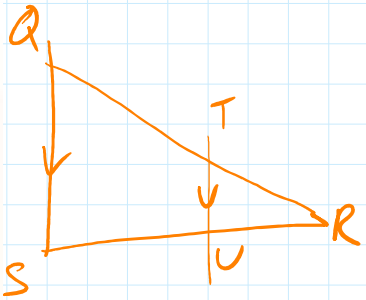
Theorem 8.6 Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.



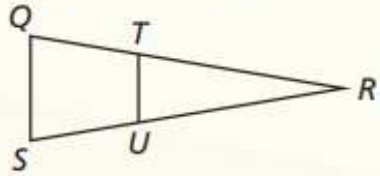
If $\overline{TU} \parallel \overline{QS}$, then $\frac{RT}{TQ} = \frac{RU}{US}$.

Proof Ex. 27, p. 451



Theorem 8.7 Converse of the Triangle Proportionality Theorem

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

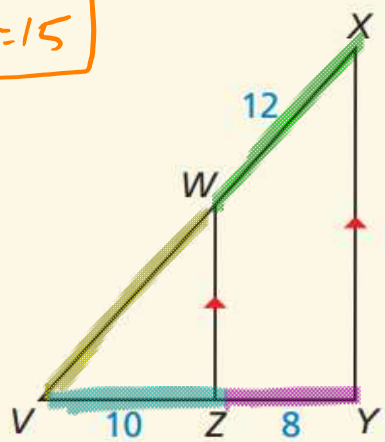


If $\frac{RT}{TQ} = \frac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.

Proof Ex. 28, p. 451

In the diagram, $\overline{WZ} \parallel \overline{XY}$, $WX = 12$, $VZ = 10$, and $ZY = 8$. What is the length of $\overline{VW}$?=

$VW = 15$



$\frac{VW}{WX} = \frac{VZ}{ZY}$

$\frac{x}{12} = \frac{10}{8}$

$\frac{x}{12} = \frac{5}{4}$

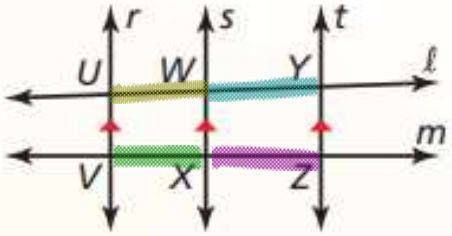
$12 \cdot 5 = 4x$

$\frac{60}{4} = \frac{4x}{4}$

$15 = x$

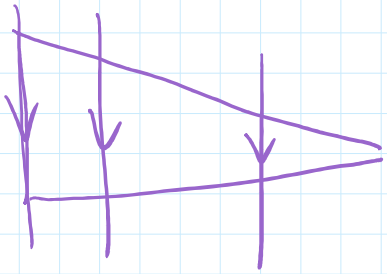
Theorem 8.8 Three Parallel Lines Theorem

If three parallel lines intersect two transversals, then they divide the transversals proportionally.

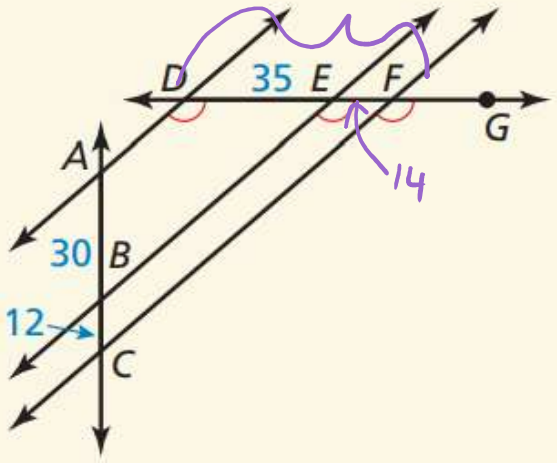


Proof Ex. 32, p. 451

$\frac{UW}{WY} = \frac{VX}{XZ}$



In the diagram, $\angle ADE$, $\angle BEF$, and $\angle CFG$ are all congruent. $AB = 30$, $BC = 12$, and $DE = 35$. Find DF.



Because $\angle$s are $\cong$ lines are $\parallel$

$$\frac{DE}{EF} = \frac{AB}{BC}$$

$$\frac{35}{x} = \frac{30}{12}$$

$$\frac{35}{x} = \frac{5}{2}$$

$$2 \cdot 35 = 5x$$

$$70 = 5x$$

$$14 = x$$

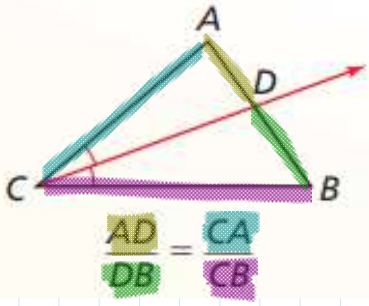
$35 + 14$

$DF = 49$

Theorem 8.9 Triangle Angle Bisector Theorem

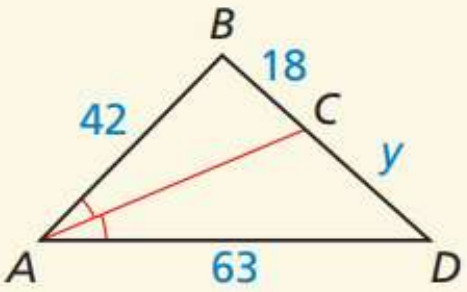
If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.

Proof Ex. 35, p. 452



$$\frac{AD}{DB} = \frac{CA}{CB}$$

In the diagram, $\angle BAC \cong \angle CAD$. Use the given lengths to find the length of $\overline{CD}$. $CD = 27$



$\frac{BC}{CD} = \frac{AB}{AD}$

$\frac{18}{y} = \frac{42}{63}$

$\frac{18}{y} = \frac{2}{3}$

$18 \cdot 3 = 2y$

$\frac{54}{2} = \frac{2y}{2}$

$27 = y$

Practice sec 8.4 pg. 450: 3-8A, 13-26A
