4.5 Dilations

Tuesday, September 5, 2017 1:18 PM

What You Will Learn

- Identify and perform dilations.

Dilations

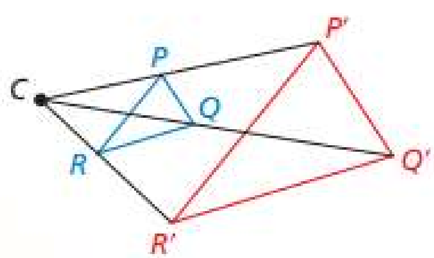
Origin

A **dilation** is a transformation in which a figure is enlarged or reduced with respect to a fixed point C called the **center of dilation** and a **scale factor** k, which is the ratio of the lengths of the corresponding sides of the image and the preimage.

$\frac{5}{1} \div \frac{1}{2} = \frac{5}{2}$

A dilation with center of dilation C and scale factor k maps every point P in a figure to a point P' so that the following are true.

- If P is the center point C, then $P = P'$.
- If P is not the center point C, then the image point P' lies on $\overrightarrow{CP}$. The scale factor k is a positive number such that $k = \frac{CP'}{CP}$. $\frac{\text{new}}{\text{old}}$
- Angle measures are preserved.



When the scale factor $k > 1$, a dilation is an **enlargement**. When $0 < k < 1$, a dilation is a **reduction**.

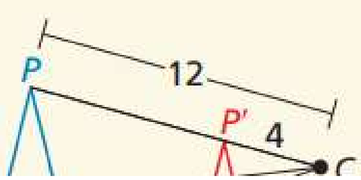
Find the scale factor of the dilation. Then tell whether the dilation is a *reduction* or an *enlargement*.

$\frac{\text{new}}{\text{old}}$

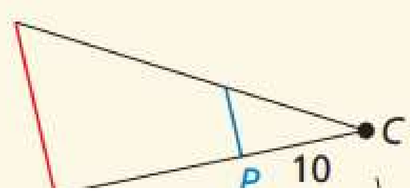
$k > 1$ = enlargement

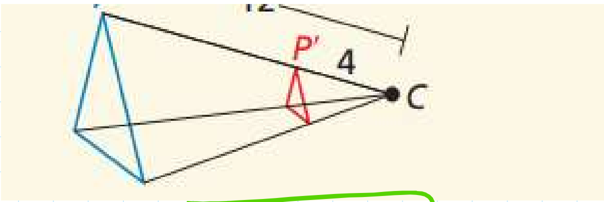
$0 < k < 1$ = reduction

a.



b.





$\frac{4}{12} = \boxed{\frac{1}{3}, \text{reduction}}$

$\frac{25}{10} = \boxed{\frac{5}{2} \text{ enlargement}}$

1. In a dilation, $CP' = 3$ and $CP = 12$. Find the scale factor. Then tell whether the dilation is a *reduction* or an *enlargement*.

$\frac{3}{12} = \boxed{\frac{1}{4}, \text{reduction}}$

$\frac{\text{new}}{\text{old}}$

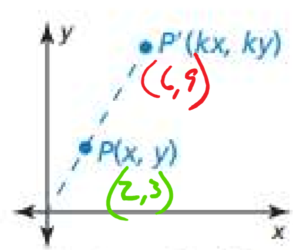
$k > 1$ = enlargement

$0 < k < 1$ = reduction

Coordinate Rule for Dilations

If $P(x, y)$ is the preimage of a point, then its image after a dilation centered at the origin $(0, 0)$ with scale factor k is the point $P'(kx, ky)$.

$k = 3$

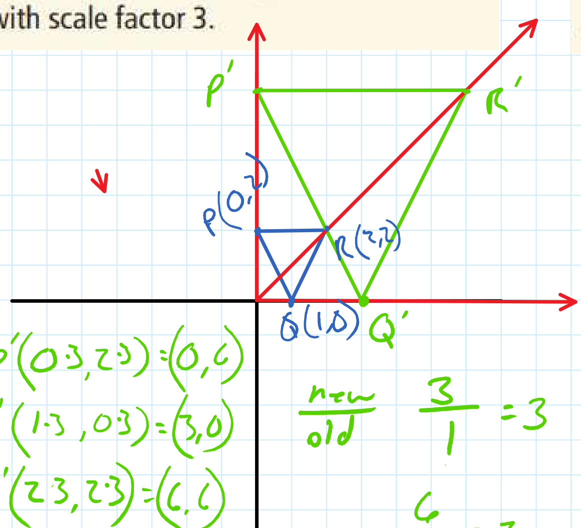


$(x, y) \rightarrow (kx, ky)$

$(2 \cdot 3, 3 \cdot 3)$

$P'(6,9)$

Graph $\triangle PQR$ with vertices $P(0, 2)$, $Q(1, 0)$, and $R(2, 2)$ and its image after a dilation with scale factor 3.



$P'(0\cdot3, 2\cdot3) = (0,6)$

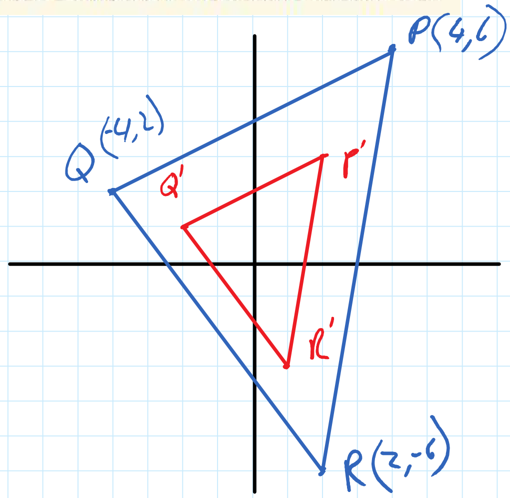
$Q'(1\cdot3, 0\cdot3) = (3,0)$

$R'(2\cdot3, 2\cdot3) = (6,6)$

$\frac{\text{new}}{\text{old}}$ $\quad \frac{3}{1} = 3$

$\frac{6}{2} = 3$

Graph $\triangle PQR$ with vertices $P(4, 6)$, $Q(-4, 2)$, and $R(2, -6)$ and its image after a dilation with scale factor 0.5.

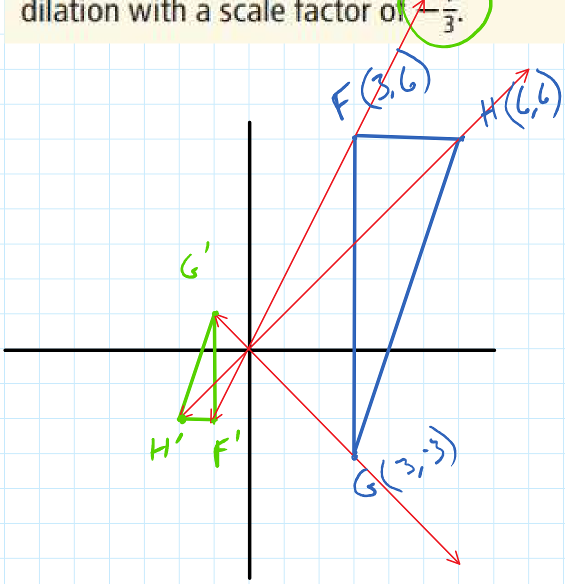


$P'(4\cdot\frac{1}{2}, 6\cdot\frac{1}{2}) = (2,3)$

$Q'(-4\cdot\frac{1}{2}, 2\cdot\frac{1}{2}) = (-2,1)$

$R'(2\cdot\frac{1}{2}, -6\cdot\frac{1}{2}) = (1,-3)$

Graph $\triangle FGH$ with vertices $F(3, 6)$, $G(3, -3)$, and $H(6, 6)$ and its image after a dilation with a scale factor of $-\frac{1}{3}$.

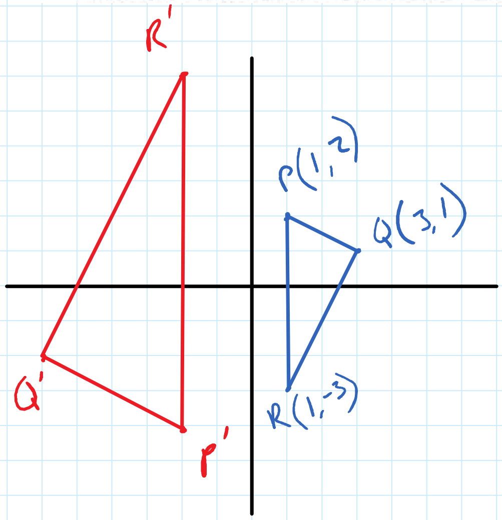


$F'(3\cdot\frac{-1}{3}, 6\cdot\frac{-1}{3}) = (-1,-2)$

$G'(3\cdot\frac{-1}{3}, -3\cdot\frac{-1}{3}) = (-1,1)$

$H'(6\cdot\frac{-1}{3}, 6\cdot\frac{-1}{3}) = (-2,-2)$

4. Graph $\triangle PQR$ with vertices $P(1, 2)$, $Q(3, 1)$, and $R(1, -3)$ and its image after a dilation with a scale factor of -2.



$P'(1 \cdot -2, 2 \cdot -2) = (-2, -4)$

$Q'(3 \cdot -2, 1 \cdot -2) = (-6, -2)$

$R'(1 \cdot -2, -3 \cdot -2) = (-2, 6)$

On the back of your log: How are dilations different from rigid transformations?

Practice sec 4.5 **pg. 212:**

1-5EO, 15-23EO