What You Will Learn

- Write conditional statements.
- Use definitions written as conditional statements.
- Write biconditional statements.

Conditional Statement

A **conditional statement** is a logical statement that has two parts, a *hypothesis p* and a *conclusion q*. When a conditional statement is written in **if-then form**, the "if" part contains the **hypothesis** and the "then" part contains the **conclusion**.

Words If p, then q. **Symbols** $p \rightarrow q$ (read as "p implies q")

hypothesis = p = "if"

"p leads to q"

conclusion = q = "then"

Use ~~(H)~~ p to identify the hypothesis and ~~(C)~~ q to identify the conclusion. Then rewrite each conditional in if-then form.

a. $\underbrace{x > 5}_{q}$ if $\underbrace{x > 3}_{p}$.

b. All members of the soccer team have practice today.

F if $x > 3$ then $x > 5$

$x = 4$?

if you are a member of the soccer team, then you have practice today.

$x = 3?$ $x = 4$

Conditional statements can be either true or false.

Negation

The **negation** of a statement is the *opposite* of the original statement. To write the negation of a statement p, you write the symbol for negation ($\sim$) before the letter. So, "not p" is written $\sim p$.

$\sim$ = "not"

Words not p **Symbols** $\sim p$

$x \approx 3.8$

Write the negation of each statement.

a. The car is white.

the car is not white

b. It is not snowing.

it is not not snowing

it is snowing

$4 \to -4$

$--4 \to 4$

Related Conditionals

Consider the conditional statement below.

Words If p, then q. **Symbols** $p \to q$

Converse To write the **converse** of a conditional statement, exchange the hypothesis and the conclusion.

Words If q, then p. **Symbols** $q \to p$

Inverse To write the **inverse** of a conditional statement, negate both the hypothesis and the conclusion.

Words If not p, then not q. **Symbols** $\sim p \to \sim q$

Contrapositive To write the **contrapositive** of a conditional statement, first write the converse. Then negate both the hypothesis and the conclusion.

Words If not q, then not p. **Symbols** $\sim q \to \sim p$

A conditional statement and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional statement are either both true or both false. In general, when two statements are both true or both false, they are called **equivalent statements**.

conditional statement $p \to q$

converse $q \to p$

inverse $\sim p \to \sim q$

contrapositive $\sim q \to \sim p$

$\neq$

$\neq$

Let p be "you are in MSHS" and let q by "you are in the USA." Write each statement in words and decide whether it is true or false.

a. The conditional statement $p \to q$.

T If you are in MSHS, then you are in the USA.

b. The conditional statement $q \to p$.

F If you are in the USA, then you are in MSHS.

c. The conditional statement $\sim p \to \sim q$.

F If you are not in MSHS, then you are not in the USA

d. The conditional statement $\sim q \to \sim p$.

T If you are not in the USA, then you are not in MSHS.

---

p in one of 50 states,

q in the USA

Using Definitions

You can write a definition as a conditional statement in if-then form or as its converse. Both the conditional statement and its converse are true for definitions. For example, consider the definition of *perpendicular lines*.

If two lines intersect to form a right angle, then they are

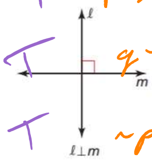
T $p \to q$ if you are in one of the 50 states then you are in the USA

You can write a definition as a conditional statement in if-then form or as its converse. Both the conditional statement and its converse are true for definitions. For example, consider the definition of *perpendicular lines*.

If two lines intersect to form a right angle, then they are **perpendicular lines**.

You can also write the definition using the converse: If two lines are perpendicular lines, then they intersect to form a right angle.

You can write "line ℓ is perpendicular to line m" as $\ell \perp m$.



T $p \to q$ if you are in one of the 50 states then you are in the USA

T $q \to p$ if you are in the USA, then you are in one of the 50 states

T $\sim p \to \sim q$ if you are not in one of the 50 states, then you aren't in the USA

T $\sim q \to \sim p$ if you aren't in the USA, then you aren't in one of the 50 states.

Biconditional Statement

When a conditional statement and its converse are both true, you can write them as a single *biconditional statement*. A **biconditional statement** is a statement that contains the phrase "if and only if."

Words p if and only if q **Symbols** $p \leftrightarrow q$

Any definition can be written as a biconditional statement.

you are in the USA if and only if you are in one of the 50 states

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Rewrite the definition of complementary angles as a single biconditional statement.
Definition: ~~If~~ two angles are complementary, ~~then~~ the sum of the measures of the angles is 90°.

two angles are complementary if and only if the sum of the measures of the angles is 90°.

10. Rewrite the definition of a right angle as a single biconditional statement.

Definition ~~If~~ an angle is a right angle, ~~then~~ its measure is 90°.

an angle is a right angle if and only if its measure is 90°.

Practice *sec 2.1* **pg. 71:**
1-3A, 5-37EO

What You Will Learn

- Use inductive reasoning.
- Use deductive reasoning.

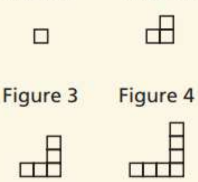
Inductive Reasoning

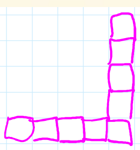
A **conjecture** is an unproven statement that is based on observations. You use **inductive reasoning** when you find a pattern in specific cases and then write a conjecture for the general case.

Conjecture is an educated guess.

Describe how to sketch the fifth figure in the pattern. Then sketch the fifth figure.

Figure 1 Figure 2 Figure 3 Figure 4

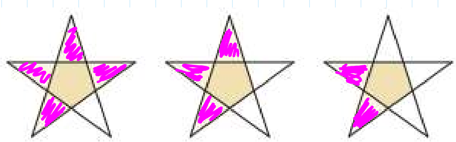




add 1 block to each "leg"

Sketch the next figure in the pattern.

2.



remove the shading from one of the points.

2,4,6,8,10,12, counting by 2's

2,4,8,16,32 multiplied by 2

2,4,16,

2,4

must have at least 3 examples to be accurate in inductive reasoning

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Counterexample

To show that a conjecture is true, you must show that it is true for all cases. You can show that a conjecture is false, however, by finding just one *counterexample*. A **counterexample** is a specific case for which the conjecture is false.

A student makes a conjecture about absolute values. Find a counterexample to disprove the student's conjecture.
Conjecture: The absolute value of the sum of two numbers is equal to the sum of the two numbers.

$|a+b| = a+b$

$a=2$, $b=4$: $|2+4| = 2+4$; $|6| = 6$; $6=6$

$a=5$, $b=15$: $|5+15| = 5+15$; $|20| = 20$; $20=20$

$a=-5$, $b=6$: $|-5+6| = -5+6$; $|1| = 1$; $1=1$

$a=-6$, $b=5$: $|-6+5| = -6+5$; $|-1| = -1$; $1 \neq -1$

Counterexample

Deductive Reasoning

Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to form a logical argument. This is different from *inductive reasoning*, which uses specific examples and patterns to form a conjecture.

Laws of Logic

Law of Detachment

If the hypothesis of a true conditional statement is true, then the conclusion is also true.

1) If you are in MSHS, then you are in the USA. You are in MSHS. You are in USA.

A: You are in the USA.

A: Not possible

Law of Syllogism

If hypothesis p, then conclusion q.
If hypothesis q, then conclusion r. — If these statements are true,

If hypothesis p, then conclusion r. ← then this statement is true.

$p \to q$
$q \to r$
$p \to r$

If a figure is a square, then it is a rectangle. You know that quadrilateral ABCD is a square. Using the law of Detachment, what statement can you make?

quad. ABCD is a rectangle

$m\angle R = 155°$

8. If $90° < m\angle R < 180°$, then $\angle R$ is obtuse. The measure of $\angle R$ is 155°. Using the Law of Detachment, what statement can you make?

$\angle R$ is obtuse

Not possible

If possible, use the law of syllogism to write a new conditional statement that follows from the pair of true statements.

~~If soccer practice is cancelled~~, then you can go to the mall after school. If it is raining today, ~~then soccer practice is cancelled~~. if it is raining, then you can go to the mall.

$p \to q$
$q \to r$
$p \to r$

If it is raining, ~~then you can go inside~~.
~~If you go inside~~, then you can play video games.
if it is raining, then you can play video games.

The table shows the sum of measures of the interior angles in various polygons. What conclusion can you make about the sum of interior angles in an *n*-sided polygon?

Polygon	Number of sides	Sum of interior angles
Triangle	3	180°
Quadrilateral	4	360°
Pentagon	5	540°
Hexagon	6	720°

Decide whether inductive reasoning or deductive reasoning is used to reach the conclusion. Explain your reasoning.

a. If the sum of the digits of a number is divisible by 3, then the number is divisible by 3. The sum of the digits of the number 147 is 12. So the number 147 is divisible by 3.

Deductive (Detachment)

b. Each time you forget to do your math homework, your parents take away your phone privileges for a day. So, the next time you forget to do your math homework, you will lose your phone privileges.

Inductive Reasoning

Practice sec 2.2 **pg.**

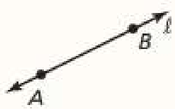
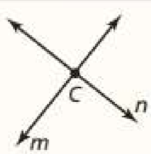
80: 1-3A,
5-25EO, 29-33EO

What You Will Learn

- Identify postulates using diagrams.
- Sketch and interpret diagrams.

Point, Line, and Plane Postulates

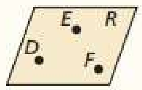
$p \to q$
$q \to p$
$\sim p \to \sim q$
$\sim q \to \sim p$
$p \leftrightarrow q$

Postulate	Example	
2.1 Two Point Postulate Through any two points, there exists exactly one line. **2.2 Line-Point Postulate** A line contains at least two points.		Through points *A* and *B*, there is exactly one line *ℓ*. Line *ℓ* contains at least two points.
2.3 Line Intersection Postulate If two lines intersect, then their intersection is exactly one point.		The intersection of line *m* and line *n* is point *C*.

Point, Line, and Plane Postulates

2.4 Three Point Postulate

Through any three noncollinear points, there exists exactly one plane.



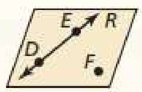
Through points D, E, and F, there is exactly one plane, plane R. Plane R contains at least three noncollinear points.

2.5 Plane-Point Postulate

A plane contains at least three noncollinear points.

2.6 Plane-Line Postulate

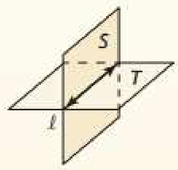
If two points lie in a plane, then the line containing them lies in the plane.



Points D and E lie in plane R, so $\overleftrightarrow{DE}$ lies in plane R.

2.7 Plane Intersection Postulate

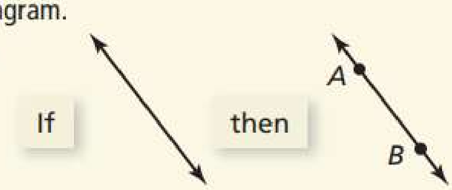
If two planes intersect, then their intersection is a line.



The intersection of plane S and plane T is line ℓ.

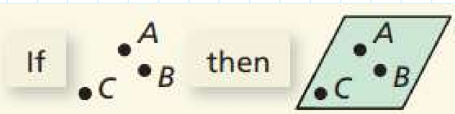
State the postulate illustrated by the diagram.

a.



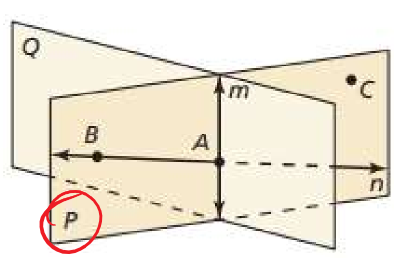
line point postulate
starts w/ line, then
adds 2 points on it.

b.



3-point post.
start w/ 3 noncollinear points
then add a plane.

Using the diagram, write an example of the Three Point Postulate (Post. 2.4)

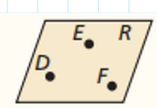


A, B, C are noncollinear points,
∴ they create plane BAC

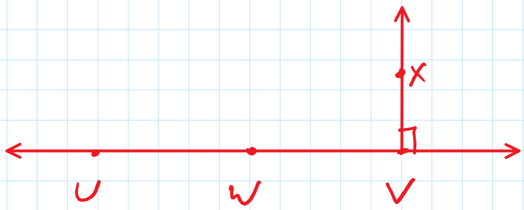
∴ = therefore

2.4 Three Point Postulate

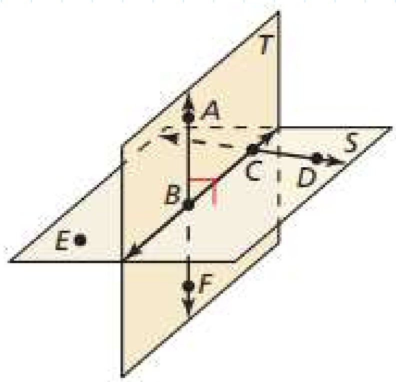
Through any three noncollinear points, there exists exactly one plane.



Sketch a diagram showing $\overrightarrow{VX}$ intersecting $\overleftrightarrow{UW}$ at V so that $\overrightarrow{VX}$ is perpendicular to $\overleftrightarrow{UW}$ and U, V, and W are collinear.



Using the diagram, which statements **cannot** be assumed from it?



- There exists a plane that contains points A, D, and E.

True, 3-point post.

- $AB = BF$.

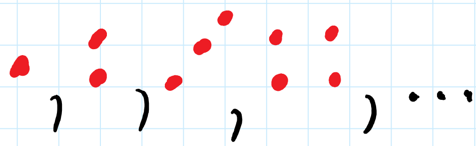
False

Practice sec 2.3 **pg. 87: 1-6A, 9-23EO**

Quiz Review

Describe the pattern and draw the next 2 in the sequence.

3, 6, 9, 12, …



(2?)

a) Rewrite the conditional statement in if then form.
b) Write the inverse.
c) Write the converse
d) Write the contrapositive
e) Check each for validity.

People who live in San Diego also live in California.

(27)

Give a counter example to each to show it is false.

1. The product of two negative numbers is always negative.

2. If complimentary angles sum to 90°, then each angle is 45°.

(27)

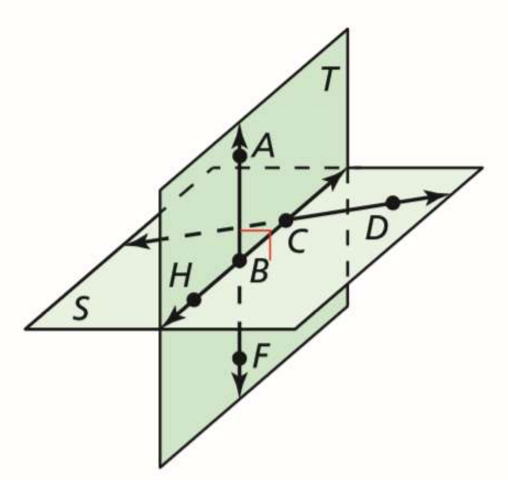
Use inductive reasoning to a) come up with a conjecture for the following. Then b) use deductive reasoning to show the conjecture to be true.

The sum of any 3 consecutive numbers…

(27)

Use the diagram to determine whether you can assume the statement is true.

A) Points A, B, and C are coplanar
B) Line AF is perpendicular to plane S.
C) Plane T is perpendicular to Line BH
D) Line CD lies on plane T
E) Point B is the midpoint for Segment AF.



13 questions!

Study and good luck!