What You Will Learn

- Write conditional statements.
- Use definitions written as conditional statements.
- Write biconditional statements.

Conditional Statement

A **conditional statement** is a logical statement that has two parts, a *hypothesis* p and a *conclusion* q. When a conditional statement is written in **if-then form**, the "if" part contains the **hypothesis** and the "then" part contains the **conclusion**.

Words If p, then q. **Symbols** $p \to q$ (read as "p implies q")

"p leads to q"

hypothesis = p = "if"

conclusion = q = "then"

Use (H) to identify the hypothesis and (C) to identify the conclusion. Then rewrite each conditional in if-then form.

a. $\underbrace{x > 5}_{C\ 'q'}$ if $\underbrace{x > 3}_{H\ p}$.

F if x>3 then x>5 x=4

b. $\overbrace{\text{All members of the soccer team}}^{H\ "p"}$ $\underbrace{\text{have practice today.}}_{C\ "q"}$

if you are a member of the soccer team, then you have practice today.

Negation

The **negation** of a statement is the *opposite* of the original statement. To write the negation of a statement p, you write the symbol for negation ($\sim$) before the letter. So, "not p" is written $\sim p$.

Words not p **Symbols** $\sim p$

$x \approx 3.8$

Write the negation of each statement.

a. The car is white.

The car is not white.

b. It is not snowing.

~~It is not not snowing~~ -3

It is snowing

Related Conditionals

Consider the conditional statement below.

Words If p, then q. **Symbols** $p \rightarrow q$

Converse To write the **converse** of a conditional statement, exchange the hypothesis and the conclusion.

Words If q, then p. **Symbols** $q \rightarrow p$

Inverse To write the **inverse** of a conditional statement, negate both the hypothesis and the conclusion.

Words If not p, then not q. **Symbols** $\sim p \rightarrow \sim q$

Contrapositive To write the **contrapositive** of a conditional statement, first write the converse. Then negate both the hypothesis and the conclusion.

Words If not q, then not p. **Symbols** $\sim q \rightarrow \sim p$

A conditional statement and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional statement are either both true or both false. In general, when two statements are both true or both false, they are called **equivalent statements**.

conditional statement $p \rightarrow q$

Converse $q \rightarrow p$

Inverse $\sim p \rightarrow \sim q$

contrapositive $\sim q \rightarrow \sim p$

$=$

$=$

Let *p* be "you are in MSHS" and let *q* by "you are in the USA." Write each statement in words and decide whether it is true or false.

a. The conditional statement $p \rightarrow q$.

T If you are in MSHS, then you are in the USA.

b. The conditional statement $q \rightarrow p$.

F If you are in the USA, then you are in MSHS.

c. The conditional statement $\sim p \rightarrow \sim q$.

F If you are not in MSHS, then you are not in the USA.

a. The conditional statement $\sim q \rightarrow \sim p$.

T If you are not in the USA, then you are not in MSHS.

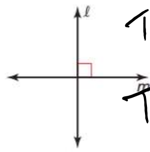
Using Definitions

You can write a definition as a conditional statement in if-then form or as its converse. Both the conditional statement and its converse are true for definitions. For example, consider the definition of *perpendicular lines*.

If two lines intersect to form a right angle, then they are **perpendicular lines**.

You can also write the definition using the converse: If two lines are perpendicular lines, then they intersect to form a right angle.

You can write "line ℓ is perpendicular to line m" as $\ell \perp m$.



$\ell \perp m$

T if $\ell \perp m$ then form right∠

T if ℓ and m form right∠, then $\ell \perp m$.

T if ℓ not $\perp$ m then they don't form right∠

T if ℓ and m do not form a right ∠, then ℓ not $\perp$ m

$\perp$

$\not\perp$

Biconditional Statement

When a conditional statement and its converse are both true, you can write them as a single *biconditional statement*. A **biconditional statement** is a statement that contains the phrase "if and only if."

Words p if and only if q **Symbols** $p \leftrightarrow q$

Any definition can be written as a biconditional statement.

$\ell \perp m$ if and only if they form right∠

$\ell \perp m \Leftrightarrow$ form right ∠

$\rightarrow$

Rewrite the definition of complementary angles as a single biconditional statement.
Definition: If two angles are complementary, then the sum of the measures of the angles is 90°.

two angles are complementary if and only if the sum of the measures of the angles is 90°.

10. Rewrite the definition of a right angle as a single biconditional statement.

Definition If $\underbrace{\text{an angle is a right angle}}_{p}$, then $\underbrace{\text{its measure is 90°}}_{q}$.

an angle is a right angle if and only if its measure is 90°

Practice *sec 2.1* **pg. 71:**
1-3A, 5-37EO