## What You Will Learn

- Use the interior angle measures of polygons.
- Use the exterior angle measures of polygons.
$>$ Use properties to find side lengths and angles of parallelograms.



## Using Interior Angle Measures of Polygons

In a polygon, two vertices that are endpoints of the same side are called consecutive vertices. A diagonal of a polygon is a segment that joins two nonconsecutive vertices.
As you can see, the diagonals from one vertex divide a polygon into triangles. Dividing a polygon with $n$ sides into $(n-2)$ triangles shows that the sum of the measures of the interior angles of a polygon is a multiple of $180^{\circ}$.

7 sides
5 D's

## Theorem 7.1 Polygon Interior Angles Theorem

The sum of the measures of the interior angles of a convex $n$-gon is $(n-2) \cdot 180^{\circ}$

$$
\begin{aligned}
& m \angle 1+m \angle 2+\cdots+m \angle n=(n-2) \cdot 180^{\circ}
\end{aligned}
$$



Proof Ex. 42 (for pentagons), p. 365

$$
\begin{aligned}
& (105-2) \cdot 180 \\
& \left(03 \cdot 180=18540^{\circ}\right.
\end{aligned}
$$

## Corollary 7.1 Corollary to the Polygon Interior Angles Theorem

The sum of the measures of the interior angles of a quadrilateral is $360^{\circ}$.

$A$ and $B$ are consecutive vertices.
Vertex $B$ has two diagonals, $\overline{B D}$ and $\overline{B E}$.

$(3-2) \cdot 180$
$1 \cdot 180$
180

Find the sum of the measures of the interior angles of the figure.


The sum of the measures of the interior angles of a convex polygon is $1800^{\circ}$. Classify the polygon by the number of sides.

$$
\begin{aligned}
& (n-2) / 80=\text { sc m of int } \angle \text { 's } \\
& \frac{(n-2) 180}{180} \frac{1800}{180} \\
& n-2=10 \\
& +2+2 \\
& n=12
\end{aligned}
$$


$(4-2)^{180}$
2.180
$360^{\circ}$

$$
\begin{gathered}
108+121+55+x=360 \\
108+180+x=360 \\
186+x=360 \\
-186-18 \\
x=72^{\circ}
\end{gathered}
$$



In an equilateral polygon, In an equiangular all sides are congruent. polygon, all angles in the interior of the polygon are congruent.


A regular polygon is a convex polygon that is both equilateral and equiangular.


A polygon is shown.

$(n-2) 180$
$(7-2) 180$
900
a. Is the polygon regular? Explain your reasoning.
No, not all Ls are congruent.
b. Find the measures of $\angle B, \angle D, \angle E$, and $\angle G$.

$$
m \angle S=125^{\circ}
$$

$100+140+160+4 x=900$
$m \angle D=125^{\circ}$
$400+4 x=900$
$-\angle E=125^{\circ}$
$\therefore \angle G=125^{\circ}$

Theorem 7.2 Polygon Exterior Angles Theorem
The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is $360^{\circ}$.

$$
m \angle 1+m \angle 2+\cdots+m \angle n=360^{\circ}
$$

Proof Ex. 51, p. 366

$n=5$

Find the value of $x$ in the diagram.

$2 x+x+67+89=340$

$3 x+15 c=3<0$

$$
\begin{gathered}
\frac{3 x}{3}=\frac{204}{3} \\
x=68
\end{gathered}
$$

A parallelogram is a quadrilateral with both pairs of opposite sides parallel. In $\square P Q R S, \overline{P Q} \| \overline{R S}$ and $\overline{Q R} \| \overline{P S}$ by definition. The theorems below describe other properties of parallelograms.


## Theorem 7.3 Parallelogram Opposite Sides Theorem

If a quadrilateral is a parallelogram, then its opposite sides are congruent.
If $P Q R S$ is a parallelogram, then $\overline{P Q} \cong \overline{R S}$ and $\overline{Q R} \cong \overline{S P}$.


Proof p. 368
Theorem 7.4 Parallelogram Opposite Angles Theorem
If a quadrilateral is a parallelogram, then its opposite angles are congruent.
If $P Q R S$ is a parallelogram, then $\angle P \cong \angle R$ and $\angle Q \cong \angle S$.


## Find the values of $x$ and $y$.



$$
2 x=54
$$

$$
x=27
$$

$$
\begin{aligned}
& 20=3 y-1 \\
& 21=3 y \\
& 7=y
\end{aligned}
$$

Theorem 7.5 Parallelogram Consecutive Angles Theorem-
If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

If $P Q R S$ is a parallelogram, then $x^{\circ}+y^{\circ}=180^{\circ}$.
Proof Ex. 38, p. 373


Theorem 7.6 Parallelogram Diagonals Theorem
If a quadrilateral is a parallelogram, then its diagonals bisect each other.
If $P Q R S$ is a parallelogram, then $\overline{Q M} \cong \overline{S M}$ and $\overline{P M} \cong \overline{R M}$.

$<3,<6$
Consoculiun Int. Bs

As shown, part of the extending arm of a desk lamp is a parallelogram. The angles of the parallelogram change as the lamp is raised and lowered. Find $m \angle B C D$ when $m \angle A D C=110^{\circ}$.


Practice sec 7.1 pg . 364: 1-25EOO, 27, 29, 37-41EO:
Sec 7.2 pg. 372:
3-19EO

## What You Will Learn

Identify and verify parallelograms.
Show that a quadrilateral is a parallelogram in the coordinate plane.
Use parallelograms in the coordinate plane.

Find the coordinates of the intersection of the diagonals of $\square A B C D$ with vertices $A(1,0), B(6,0), C(5,3)$, and $D(0,3)$.

$$
\begin{aligned}
& \left(\frac{0+6}{2}, \frac{3+0}{2}\right)\left(3, \frac{3}{2}\right) \\
& \left(\frac{1+5}{2}, \frac{0+3}{2}\right) \\
& \left(\frac{6}{2}, \frac{3}{2}\right)=\left(3, \frac{3}{2}\right)
\end{aligned}
$$



Find the coordinates of the intersection of the diagonals of $\square S T U V$ with vertices $S(-2,3), T(1,5), U(6,3)$, and $V(3,1)$.


Three vertices of $\square D E F G$ are $D(-1,4)$,
$E(2,3)$, and $F(4,-2)$. Find the coordinates of vertex $G$.

$$
G(1,-1)
$$



Three vertices of $\square A B C D$ are $A(2,4), B(5,2)$, and $C(3,-1)$. Find the coordinates of vertex $D$.


## Theorem 7.7 Parallelogram Opposite Sides Converse

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
If $\overline{A B} \cong \overline{C D}$ and $\overline{B C} \cong \overline{D A}$, then $A B C D$ is a parallelogram.


## Theorem 7.8 Parallelogram Opposite Angles Converse

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $A B C D$ is a parallelogram.

Proof Ex. 39, p. 383


In quadrilateral $A B C D, A B=B C$ and $C D=A D$. Is $A B C D$ a parallelogram? Explain your reasoning.

$$
N_{0} \text { opposite sides ane rot }
$$

congruent.


Finding Side Lengths of a Parallelogram

For what values of $x$ and $y$ is quadrilateral STUV a parallelogram?


$$
\begin{gathered}
24-x=x+C \\
24=2 x+C \\
18=2 x \\
9=x
\end{gathered}
$$

## Theorem 7.9 Opposite Sides Parallel and Congruent Theorem

If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.
If $\overline{B C} \| \overline{A D}$ and $\overline{B C} \cong \overline{A D}$, then $A B C D$ is a parallelogram.


Proof Ex. 40, p. 383

## Theorem 7.10 Parallelogram Diagonals Converse

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
If $\overline{B D}$ and $\overline{A C}$ bisect each other, then $A B C D$ is a parallelogram.


For what value of $x$ is quadrilateral CDEF a parallelogram?


## Ways to Prove a Quadrilateral Is a Parallelogram

| 1. Show that both pairs of opposite sides are parallel. (Definition) |
| :--- |
| 2. Show that both pairs of opposite sides are congruent. <br> (Parallelogram Opposite Sides Converse) <br> 3. Show that both pairs of opposite angles are congruent. <br> (Parallelogram Opposite Angles Converse) <br> 4. Show that one pair of opposite sides are congruent and parallel. <br> (Opposite Sides Parallel and Congruent Theorem) <br> 5. Show that the diagonals bisect each other. <br> (Parallelogram Diagonals Converse) |

Show that quadrilateral $A B C D$ is a parallelogram.

$$
m \overline{A B}=\frac{2}{2}=1
$$



$$
\begin{aligned}
& m \overline{C D}=\frac{2}{2}=1 \\
& \text { m } \overline{A D}=\frac{-2}{4}=-\frac{1}{2} \\
& -\overline{C B}=\frac{-2}{4}=-\frac{1}{2}
\end{aligned}
$$

yes, ABCD is sparallologeen

Show that quadrilateral $A B C D$ is a parallelogram.


$$
\begin{aligned}
& m \overline{A B}=\frac{2}{5} \\
& \sim \overline{C D}=\frac{2}{5} \\
& m \overline{B C}=\frac{-3}{3}=-1 \\
& m \overline{A D}=\frac{-3}{2}=-1
\end{aligned}
$$

> Practice sec 7.2 pg . $372: 25-27 \mathrm{~A}, 29 ;$ sec $7.3 \mathrm{pg} .381: 1$, $3-9 \mathrm{~A}, 11-19 \mathrm{EO}$

## What You Will Learn

- Use properties of special parallelograms.
- Use properties of diagonals of special parallelograms.
- Use coordinate geometry to identify special types of parallelograms.


## Rhombuses, Rectangles, and Squares



## A rhombus is a

 parallelogram with four congruent sides.

A rectangle is a parallelogram with four right angles.


A square is a parallelogram with four congruent sides and four right angles.

In an equilateral polygon, In an equiangular all sides are congruent.
polygon, all angles in the
interior of the polygon are congruent.


A regular polygon is a convex polygon that is both equilateral and equiangular.



For any rhombus $Q R S T$, decide whether the statement is always or sometimes true. Draw a diagram and explain your reasoning.
a. $\angle Q \cong \angle S$


A lugs True beaus
Rhombus' s. Parllibgres
and opp. $\angle$ s. $\cong$
b. $\angle Q \cong \angle R$


Solis True becker
uh. consoutio <s... $\cong$
the RLahsberus: sum
sal not all Rhombs' sea squaws

Classify the special quadrilateral. Explain your reasoning.

Rhombus


## Theorem 7.11 Rhombus Diagonals Theorem

A parallelogram is a rhombus if and only if its diagonals are perpendicular.
$\square A B C D$ is a rhombus if and only if $\overline{A C} \perp \overline{B D}$.
Proof p. 390; Ex. 72, p. 395


## Theorem 7.12 Rhombus Opposite Angles Theorem

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.
$\square A B C D$ is a rhombus if and only if $\overline{A C}$ bisects $\angle B C D$ and $\angle B A D$, and $\overline{B D}$ bisects $\angle A B C$ and $\angle A D C$.


Find the measures of the numbered angles in rhombus $A B C D$.

$$
\begin{aligned}
& m \angle 1=90^{\circ} \quad 3 C 0=244+2 x \\
& m \angle 2=61^{\circ} \\
& m \angle 3=61^{\circ}
\end{aligned}
$$

Theorem 7.13 Rectangle Diagonals Theorem
A parallelogram is a rectangle if and only if its diagonals are congruent.
$\square A B C D$ is a rectangle if and only if $\overline{A C} \cong \overline{B D}$.
Prof Fix 87 and 88 n 396


In rectangle $A B C D, A C=7 x-15$ and $B D=2 x+25$. Find the lengths of the diagonals of $A B C D$.


$$
\begin{gathered}
2 x+25=7 x-15 \\
+15=15 \\
2 x+40=7 x \\
-2 x=-2 x \\
40=5 x \\
8=x
\end{gathered}
$$

$$
\begin{aligned}
& 2 x+75 ; x=8 \\
& 2.8+25 \\
& 41
\end{aligned}
$$

Decide whether $\square A B C D$ with vertices

$$
A(-2,3), B(2,2), C(1,-2) \text {, and }(-3,-1)
$$

is a rectangle, a rhombus, (or a square.
$m \overline{A B}=\frac{-1}{4}$
$m \overline{C D}=\frac{-1}{4}$
$m \overline{A D}=\frac{4}{1}$
$m \overline{B C}=\frac{4}{1}$
m $\overline{B D}=\frac{3}{5}$
~ $\overline{A C}=\frac{-5}{3}$


Practice sec 7.4 pg .

$$
\begin{aligned}
& \text { 393: } 1-3 A, \\
& 7-21 E O O, 23-35 E O, \\
& 43-57 E O O
\end{aligned}
$$

## What You Will Learn

- Use properties of trapezoids.
- Use the Trapezoid Midsegment Theorem to find distances.
- Use properties of kites.
- Identify quadrilaterals.


## Using Properties of Trapezoids

A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides are the bases.

Base angles of a trapezoid are two consecutive angles whose common side is a base. A trapezoid has two pairs of base angles. For example, in trapezoid $A B C D, \angle A$ and $\angle D$ are one pair of base angles, and $\angle B$ and $\angle C$ are the second pair. The nonparallel sides are the legs of the trapezoid.

If the legs of a trapezoid are congruent, then the trapezoid is an isosceles trapezoid.


Show that $A B C D$ is a trapezoid and decide $m \overline{B C}=\frac{-1}{3}$ whether it is isosceles.


$$
\begin{aligned}
& \text { - } \overline{A D}=\frac{-2}{6}=\frac{-1}{3} \text { establislos } \\
& m \overline{A B}=\frac{4}{2}=2 \\
& \text { a } \overline{C D}=\frac{-5}{1}=-5, \downarrow \\
& \begin{array}{l}
D=\sqrt{\left(x_{2}-x\right)^{2}+\left(y_{2}-y_{1}\right)^{2}} \text { AB }=\frac{\sqrt{(-2-0)^{2}+(-173)^{2}}}{\sqrt{(-2)^{2}+(-4)^{2}}} \\
a^{2}+b^{2}=c^{2}
\end{array} \\
& a^{2}+x^{2}-c^{2} \\
& \longdiv { 1 . 1 1 1 } \\
& 20
\end{aligned}
$$



213
$a^{2}+b^{2}=c^{2}$
$1^{2}+5^{2}=c^{2}$
$1+25=c^{2}$
$\sqrt{2 c}=c$


Theorem 7.14 Isosceles Trapezoid Base Angles Theorem
If a trapezoid is isosceles, then each pair of base angles is congruent.
If trapezoid $A B C D$ is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.

Proof Ex. 39, p. 405


Theorem 7.15 Isosceles Trapezoid Base Angles Converse
If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.
If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$ ), then trapezoid $A B C D$ is isosceles.

Proof Ex. 40, p. 405


Theorem 7.16 Isosceles Trapezoid Diagonals Theorem
A trapezoid is isosceles if and only if its diagonals are congruent.
Trapezoid $A B C D$ is isosceles if and only if $\overline{A C} \cong \overline{B D}$.

Proof Ex. 51, p. 406


## Theorem 7.17 Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base, and its length is one-half the sum of the lengths of the bases.
If $\overline{M N}$ is the midsegment of trapezoid $A B C D$, then $\overline{M N}\|\overline{A B}, \overline{M N}\| \overline{D C}$, and $M N=\frac{1}{2}(A B+C D)$.
Proof Ex. 49, p. 406


In the diagram, $\overline{M N}$ is the midsegment of trapezoid $P Q R S$. Find $M N$.


$$
13.5+18.9=\mathrm{mN}
$$



$$
\begin{aligned}
& \frac{13.5+18.9}{2}=\mathrm{mN} \\
& \frac{32.4}{2}=16.2 \mathrm{in}
\end{aligned}
$$

Find the length of midsegment $\overline{Y Z}$ in trapezoid PQRS.

$4^{2}+4^{2}=c^{2}$ $16+16=c^{2}$ $\sqrt{32}=c$ $4 \sqrt{2}=c$
$2^{2}+z^{2}=c^{2}$
$4+4=c^{2}$
$\sqrt{8}=c$
$2 \sqrt{2}=c$



$3 c+3 c=c^{2}$
$\sqrt{72}=c$
$6 \sqrt{2}=c$
$2 x+6 x$
$\frac{2 \sqrt{2}+6 \sqrt{2}}{2}$
$1^{2}+1^{2}=c^{2}$ $1+1=c^{2}$ $\sqrt{2}=c \approx 1.414 \ldots$.

## Using Properties of Kites

A kite is a quadrilateral that has two pairs of consecutive congruent sides, put opposite sides are not congruent.


## Theorem 7.18 Kite Diagonals Theorem

If a quadrilateral is a kite, then its diagonals are perpendicular.
If quadrilateral $A B C D$ is a kite, then $\overline{A C} \perp \overline{B D}$.
Proof p. 401


## Theorem 7.19 Kite Opposite Angles Theorem

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral $A B C D$ is a kite and $\overline{B C} \cong \overline{B A}$, then $\angle A \cong \angle C$ and $\angle B \nRightarrow \angle D$.


Find $m \angle C$ in the kite shown.
$80+50+2 \alpha=366$
$(n-2)^{180}$
$(4-2)^{180}$
360
$130+2 \alpha=3<0$
$-130-136$
$2 \alpha=236$
$\alpha=115$

$m \angle C=115^{\circ}$

Give the most specific name for the quadrilateral. Explain your reasoning.
8.

kite
9.



Trapezoid
Quaderlataerl
Quedicideral Parsllalogan R La bes Rectrojle Square
Tripazoid Isocales Trepraid Kite

## Practice sec 7.5 pg . <br> 403: 1-17EO, <br> 21-33EO

$\square$

## Chapter 7 Review

## Classifications of Quadrilaterals



Use the picture of the regular convex polygon to answer the following questions. Make sure to show your work.

Use the picture of the regular convex polygon to answer the following questions. Make sure to show your work.

1. What is the sum of the interior angles?
$(n-2) 180(7-2) 180$
$900^{\circ}$
2. What is the sum of the exterior angles?
$360^{\circ}$
3. What is the measure of each interior angle?


$$
\frac{900^{\circ}}{7} \approx 128.4^{\circ}
$$

4. What is the measure of each exterior angle?

$$
\frac{360^{\circ}}{7} \approx 51.6^{\circ}
$$

Write an equation and then solve to find the value of $x$.


$$
\begin{aligned}
143+2 x+152+116+125+140 & +139+x=1080 \\
815+3 x & =1080 \\
\frac{3 x}{3} & =\frac{2 c 5}{3} \\
x & =\frac{2 c 5^{\circ}}{3} \approx 88.3^{\circ}
\end{aligned}
$$

Find the measure of each exterior angle of a regular polygon in which the sum of the measures of the interior angles is $1980^{\circ}$. Show your work.

$$
\begin{gathered}
\frac{(n-2) 180}{180}=\frac{1980}{180} \\
n-2=11 \\
+2+2 \\
n=13
\end{gathered}
$$

$$
\begin{aligned}
& \frac{360^{\circ}}{13}=\text { ext. angle mersin } \\
& 27.7 \text { pant } \\
& \frac{360^{\circ}}{13} \approx 27.7^{\circ}
\end{aligned}
$$

PQRS is a parallelogram. Use the picture to find the indicated values or measures. Show your work.

Find $y .=7$

Find $x .=27$


$$
\begin{array}{cc}
3 y-1=20 & \frac{2 x}{2}=\frac{54}{2} \\
3 y=21 & x=27 \\
y=7 & 54+x=180 \\
x=126
\end{array}
$$

Three vertices of $\square A B C D$ are $A(2,4), B(5,2)$, and $C(3,-1)$. Find the coordinates of vertex $D$.


Find the values of $m$ and $n$ that make the quadrilateral a parallelogram.


$$
\begin{gathered}
n+70=180 \\
n=110
\end{gathered}
$$

$$
\begin{aligned}
70+70+2 n & =360 \\
140+2 n & =350 \\
2 n & =220 \\
n & =110
\end{aligned}
$$

Use mathematical computations to show that quadrilateral WXYZ is a parallelogram.

$$
\begin{array}{r|r}
W(-2,5), X(2,5), Y(4,0), Z(0,0) & \text { m } \overline{W X}=\frac{0}{4}=0 \\
\text { w }(2,5) & x(2,5)
\end{array}
$$



Circle all names that apply to the given shape.


Kite
trapezoid

The diagonals of rhombus DEFG intersect at
$P$. Given that $P E=4$ find the indicated measures.

$\mathrm{m} \angle \mathrm{DEG}=59^{\circ}$
$\mathrm{m} \angle E D G=|2|^{\circ}$
$\cdots$ O

$$
D+5 S=180
$$


$\mathrm{m} \angle E D G=|2|^{\circ}$

$$
\mathrm{GE}=
$$

$$
\begin{gathered}
D+5 S=180 \\
D=121
\end{gathered}
$$



Determine if the given points represent the vertices of a trapezoid. If so, determine whether it is isosceles or not.

$$
A(-5,6), B(4,9), C(4,4) \text {, and } D(-2,2)
$$



$$
\text { 2) } \overline{A B}=\frac{3}{9}=\frac{1}{3}
$$

$$
n \overline{C D}=\frac{2}{6}=\frac{1}{3}
$$

Trap.

$$
\sim \overline{A D}=\frac{-4}{3}
$$

$$
\angle \overline{B C}=\frac{-5}{0}=\text { undef. }
$$

$$
\begin{aligned}
B C & =5-1 \\
A D & =5 \\
4^{2}+3^{2} & =c^{2} \\
16+9 & =c^{2} \\
25 & =c^{2} \\
5 & =c
\end{aligned}
$$

I scales Tres

Find the length of the 2 bases of the trapezoid. Show your work.


$$
\begin{aligned}
& \frac{b+b_{2}}{2}=\text { nide gmat } \left\lvert\, \begin{array}{l}
3 x+2 ; x=c \\
3-6+2 \\
18+2 \\
20
\end{array}\right.
\end{aligned}
$$



In kite $A B C D, m \angle C D B=14^{\circ}$, and $m \angle C B D=46^{\circ}$
Find the indicated measure.

$60+C=80$

$$
C=120
$$

The End!

23 total questions
Notecard allowed!

