8.1 similar polygons

Tuesday, September 5, 2017 1:18 PM

# What You Will Learn

- Use similarity statements.
- Find corresponding lengths in similar polygons.
- Find perimeters and areas of similar polygons.
- Decide whether polygons are similar.

$\triangle ABC \cong \triangle XYZ$

$\angle A \cong \angle X$ $\quad \overline{AB} \cong \overline{XY}$

$\angle B \cong \angle Y$ $\quad \overline{BC} \cong \overline{YZ}$

$\angle C \cong \angle Z$ $\quad \overline{AC} \cong \overline{XZ}$

$\cong$ means same size same shape

$\sim$ means same shape

$\triangle ABC \sim \triangle XYZ$

$\angle A \cong \angle X$ $\quad \overline{AB} \sim \overline{XY}$

$\angle B \cong \angle Y$ $\quad \overline{BC} \sim \overline{YZ}$

$\angle C \cong \angle Z$ $\quad \overline{AC} \sim \overline{XZ}$

## Corresponding Parts of Similar Polygons

In the diagram below, $\triangle ABC$ is similar to $\triangle DEF$. You can write "$\triangle ABC$ is similar to $\triangle DEF$" as $\triangle ABC \sim \triangle DEF$. A similarity transformation preserves angle measure. So, corresponding angles are congruent. A similarity transformation also enlarges or reduces side lengths by a scale factor $k$. So, corresponding side lengths are proportional.


How we usually use k

### Corresponding angles

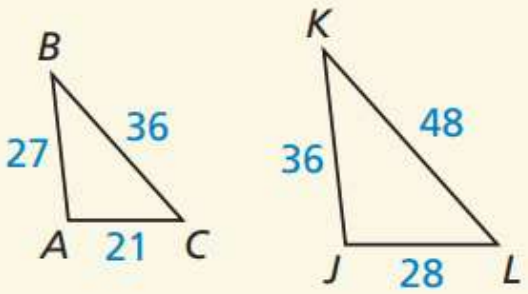
$\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$

### Ratios of corresponding side lengths

$$\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA} = k$$

How we usually find k

In the diagram, $\triangle ABC \sim \triangle JKL$.




**a.** Find the scale factor from $\triangle ABC$ to $\triangle JKL$.

$\frac{AB}{JK} = \frac{27}{36} = \boxed{\frac{3}{4}}$

$\frac{BC}{KL} = \frac{36}{48} = \frac{18}{24} = \boxed{\frac{3}{4}}$ $\frac{KL}{BC} = \frac{48}{36} = \frac{4}{3}$

$\frac{AC}{JL} = \frac{21}{28} = \boxed{\frac{3}{4}}$

$K = \frac{3}{4}$

**b.** List all pairs of congruent angles.

$\angle A \cong \angle J$ $\angle B \cong \angle K$ $\angle C \cong \angle L$

**c.** Write the ratios of the corresponding side lengths in a *statement of proportionality.*

$\frac{AB}{JK} = \frac{BC}{KL} = \frac{AC}{JL}$

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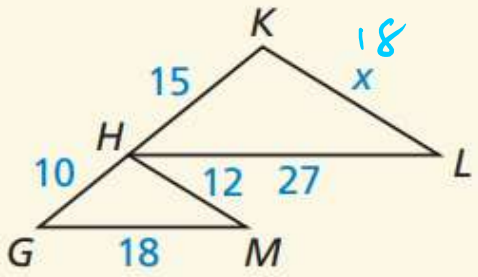
## Corresponding Lengths in Similar Polygons

If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the scale factor of the similar polygons.

In the diagram, $\triangle GHM \sim \triangle HKL$. Find the value of $x$.




$\frac{GH}{HK} = \frac{HM}{KL} = \frac{GM}{HL}$

$\frac{10}{15} = \frac{12}{x} \rightarrow \frac{2}{3} = \frac{12}{x}$

$15 \cdot 12 = 10x$ $3 \cdot 12 = 2x$

$\frac{180}{10} = \frac{10x}{10}$ $\frac{36}{2} = \frac{2x}{2}$

$18 = x$ $18 = x$

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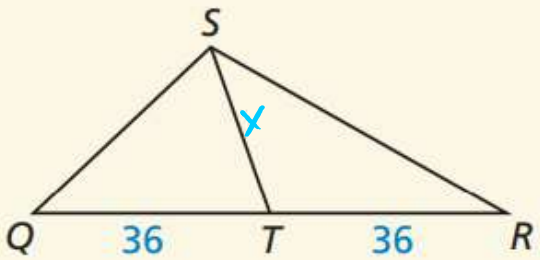
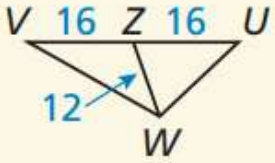
In the diagram, $\triangle UVW \sim \triangle QRS$. Find the length of the median $\overline{ST}$. = 27




$\frac{12}{x} = \frac{32}{72} = \frac{16}{36} = \frac{8}{18} = \frac{4}{9}$

$32x = 12 \cdot 72$

$\frac{32x}{32} = \frac{864}{32}$

$x = 27$

$\frac{12}{27} = \frac{4}{9}$

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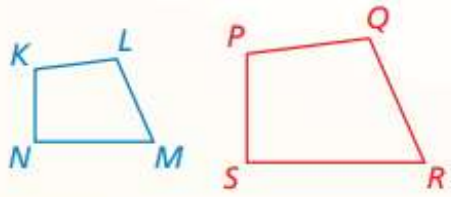
**Theorem 8.1 Perimeters of Similar Polygons**

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.

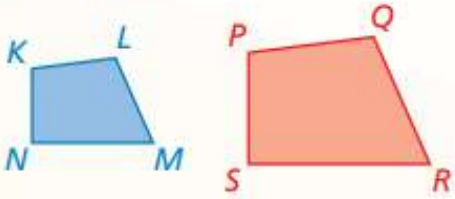



If $KLMN \sim PQRS$, then $\frac{PQ + QR + RS + SP}{KL + LM + MN + NK} = \frac{PQ}{KL} = \frac{QR}{LM} = \frac{RS}{MN} = \frac{SP}{NK}$.

$\left(\frac{1}{2}\right)^2 = \frac{1^2}{2^2} = \frac{1}{4}$

**Theorem 8.2 Areas of Similar Polygons**

If two polygons are similar, then the ratio of their areas is equal to the squares of the ratios of their corresponding side lengths.




If $KLMN \sim PQRS$, then $\frac{\text{Area of } PQRS}{\text{Area of } KLMN} = \left(\frac{PQ}{KL}\right)^2 = \left(\frac{QR}{LM}\right)^2 = \left(\frac{RS}{MN}\right)^2 = \left(\frac{SP}{NK}\right)^2$.

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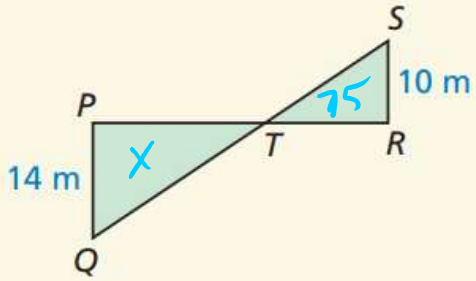
In the diagram, $\triangle PQT \sim \triangle RST$, and the area of $\triangle RST$ is 75 square meters. Find the area of $\triangle PQT$.




147 $m^2$

$\frac{147}{75} = \left(\frac{14}{10}\right)^2$

$\frac{147}{75} = \frac{49}{25}$

$\frac{X}{75} = \left(\frac{PQ}{RS}\right)^2$

$\frac{X}{75} = \left(\frac{14}{10}\right)^2 \rightarrow \frac{X}{75} = \left(\frac{14}{10}\right)^2$

$\frac{X}{75} = \left(\frac{7}{5}\right)^2$

$\frac{X}{75} = \frac{196}{100}$

$\frac{X}{75} = \frac{49}{25}$

$75 \cdot 196 = 100X$

$\frac{14700}{100} = \frac{100X}{100}$

$X = 147$

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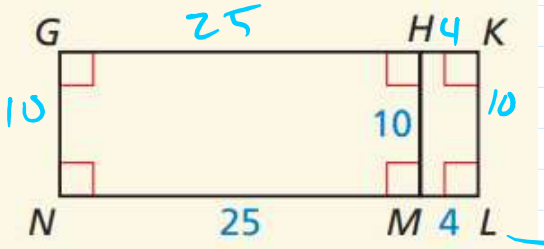
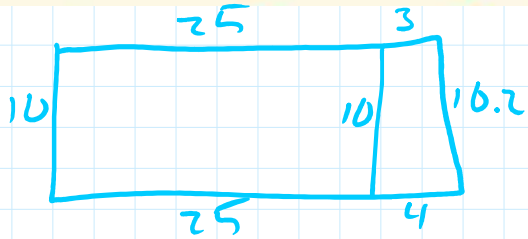
Decide whether *GNMH* and *MLKH* are similar. Explain your reasoning.




$\frac{GN}{ML} = \frac{NM}{LK} = \frac{MH}{KH} = \frac{GH}{MH}$

$\frac{10}{4} = \frac{25}{10} = \frac{10}{4} = \frac{25}{10}$

$\frac{5}{2} = \frac{5}{2} = \frac{5}{2} = \frac{5}{2}$

Similar

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Practice *sec 8.1* **pg. 423: 1-15EO, 19-31EO**