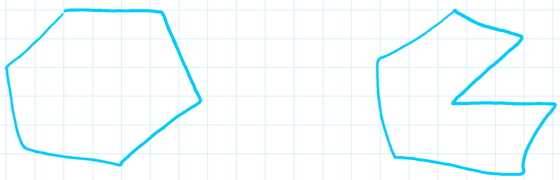
What You Will Learn

- Use the interior angle measures of polygons.
- Use the exterior angle measures of polygons.
- Use properties to find side lengths and angles of parallelograms.

Convex



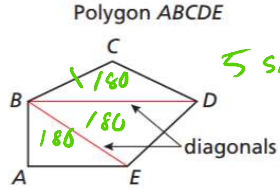
Concave

Using Interior Angle Measures of Polygons

In a polygon, two vertices that are endpoints of the same side are called *consecutive vertices*. A **diagonal** of a polygon is a segment that joins two nonconsecutive vertices.

As you can see, the diagonals from one vertex divide a polygon into triangles. Dividing a polygon with n sides into $(n-2)$ triangles shows that the sum of the measures of the interior angles of a polygon is a multiple of 180°.

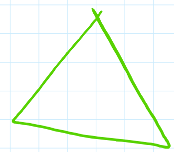
Polygon *ABCDE*

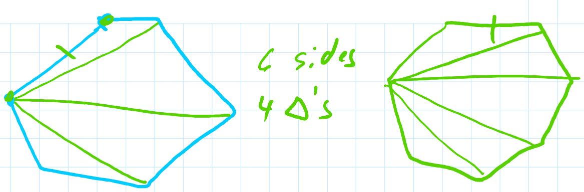


A and *B* are consecutive vertices.
Vertex *B* has two diagonals, $\overline{BD}$ and $\overline{BE}$.

5 sides 3 Δ's

180 + 180 + 180 = 540°





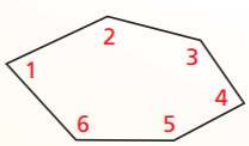
6 sides
4 Δ's

7 sides
5 Δ's

Theorem 7.1 Polygon Interior Angles Theorem

The sum of the measures of the interior angles of a convex n-gon is $(n-2) \cdot 180°$.

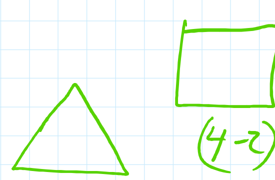
$$m\angle 1 + m\angle 2 + \cdots + m\angle n = \boxed{(n-2) \cdot 180°}$$



Proof Ex. 42 (for pentagons), p. 365

$n = 6$

$(6-2)180$
$4 \cdot 180$
$\boxed{720°}$



$(3-2) \cdot 180$
$1 \cdot 180$
180

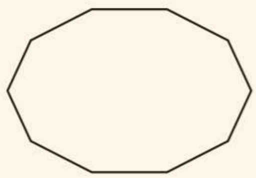
$(4-2)180$
$2 \cdot 180$
360

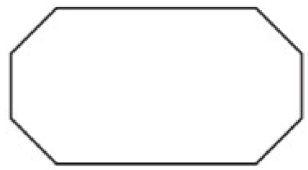
$(105-2) \cdot 180$
$103 \cdot 180 = 18540°$

Corollary 7.1 Corollary to the Polygon Interior Angles Theorem

The sum of the measures of the interior angles of a quadrilateral is 360°.

Find the sum of the measures of the interior angles of the figure.





The sum of the measures of the interior angles of a convex polygon is 1800°. Classify the polygon by the number of sides.

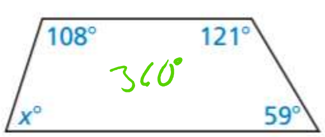
$(n-2)180 =$ sum of int $\angle$'s

$\frac{(n-2)\cancel{180}}{\cancel{180}} = \frac{1800}{180}$

$n-2 = 10$

$+2 \quad +2$

$\boxed{n = 12}$



$(4-2)180$

$2 \cdot 180$

$360°$

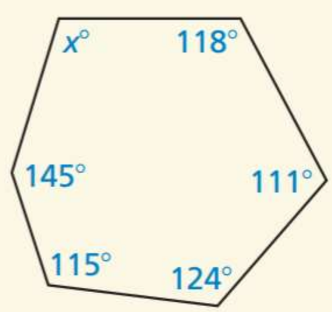
$108+121+59+x=360$

$108+180+x=360$

$288+x=360$

$-288 \quad -288$

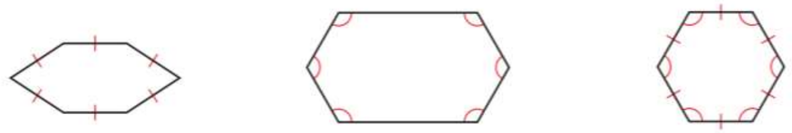
$\boxed{x=72°}$

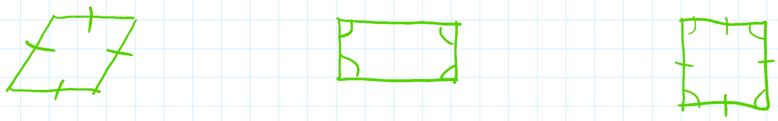


In an **equilateral polygon**, all sides are congruent.

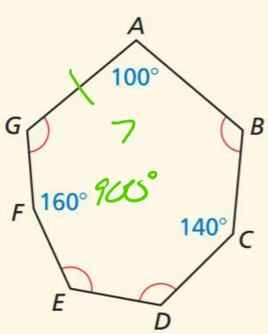
In an **equiangular polygon**, all angles in the interior of the polygon are congruent.

A **regular polygon** is a convex polygon that is both equilateral and equiangular.





A polygon is shown.



a. Is the polygon regular? Explain your reasoning.

No, not all ∠s are congruent.

b. Find the measures of ∠B, ∠D, ∠E, and ∠G.

$(n-2)180$
$(7-2)180$
900

$100+140+160+4x=900$
$400+4x=900$
$4x=500$
$x=125$

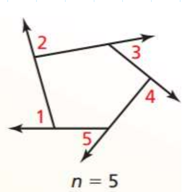
$m\angle B = 125°$
$m\angle D = 125°$
$m\angle E = 125°$
$m\angle G = 125°$

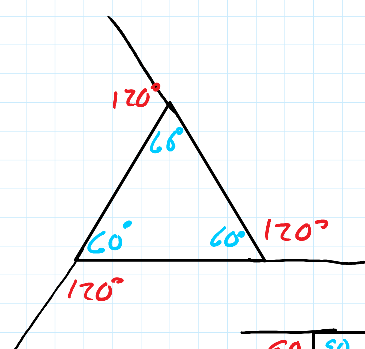
Theorem 7.2 Polygon Exterior Angles Theorem

The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360°.

$m\angle 1 + m\angle 2 + \cdots + m\angle n = 360°$

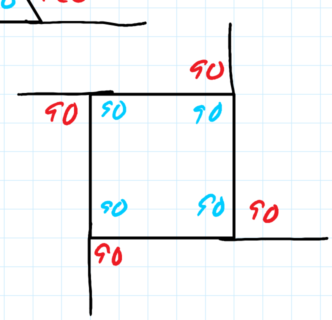
Proof Ex. 51, p. 366



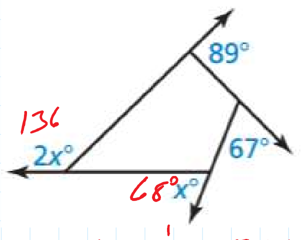


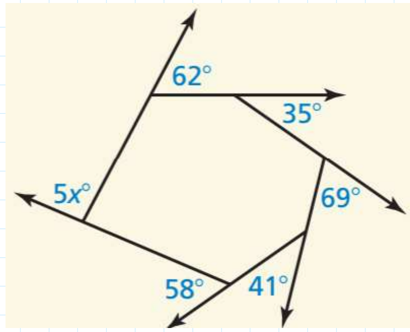
360°

$90 \cdot 4 = 360°$



Find the value of x in the diagram.





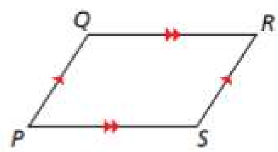
$2x + x + 67 + 89 = 360$

$3x + 156 = 360$

$\frac{3x}{3} = \frac{204}{3}$

$x = 68$

A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel. In $\square PQRS$, $\overline{PQ} \parallel \overline{RS}$ and $\overline{QR} \parallel \overline{PS}$ by definition. The theorems below describe other properties of parallelograms.

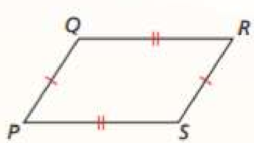


Theorem 7.3 Parallelogram Opposite Sides Theorem

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

If $PQRS$ is a parallelogram, then $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$.

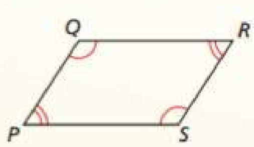
Proof p. 368



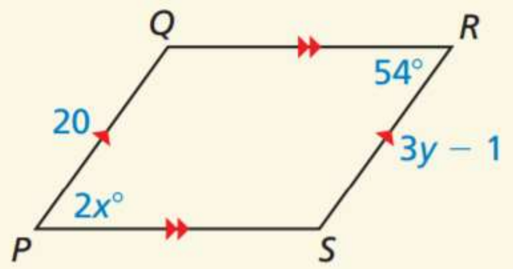
Theorem 7.4 Parallelogram Opposite Angles Theorem

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

If $PQRS$ is a parallelogram, then $\angle P \cong \angle R$ and $\angle Q \cong \angle S$.



Find the values of x and y.



$2x = 54$

$\boxed{x = 27}$

$20 = 3y - 1$

$21 = 3y$

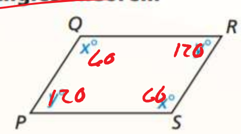
$\boxed{7 = y}$

Theorem 7.5 Parallelogram Consecutive Angles Theorem

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

If *PQRS* is a parallelogram, then $x° + y° = 180°$.

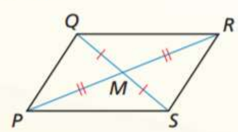
Proof Ex. 38, p. 373

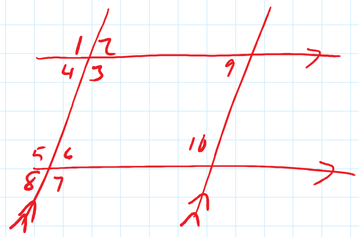


Theorem 7.6 Parallelogram Diagonals Theorem

If a quadrilateral is a parallelogram, then its diagonals bisect each other.

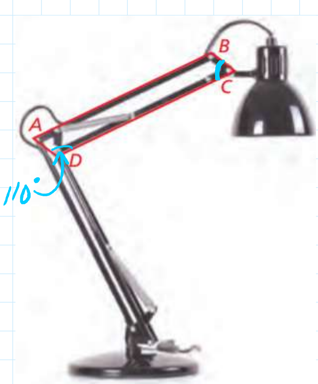
If *PQRS* is a parallelogram, then $\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$.





$\angle 3, \angle 6$ Consecutive Int. $\angle$s

As shown, part of the extending arm of a desk lamp is a parallelogram. The angles of the parallelogram change as the lamp is raised and lowered. Find $m\angle BCD$ when $m\angle ADC = 110°$.



$110 + C = 180$

$-110 \quad -110$

$C = 70$

$\boxed{m\angle BCD = 70°}$

Practice sec 7.1 **pg. 364: 1-25EOO, 27, 29, 37-41EO;**
Sec 7.2 **pg. 372: 3-19EO**