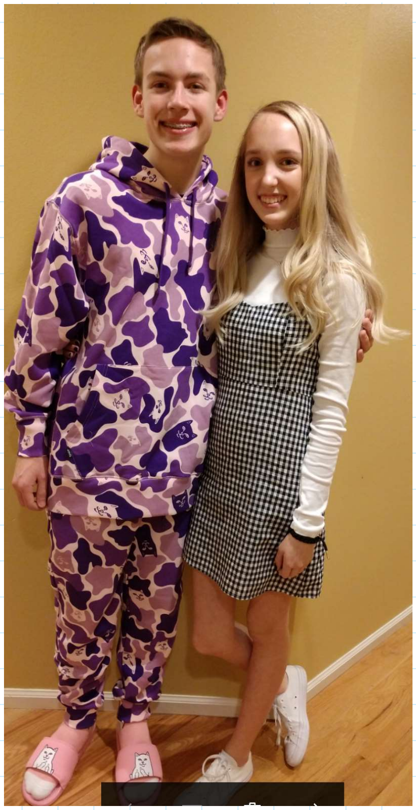
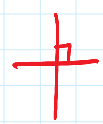
6.1 bisectors

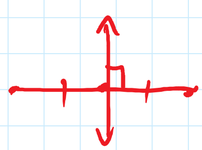
Saturday, January 7, 2017 9:42 AM



What You Will Learn

- Use perpendicular bisectors to find measures.
- Use angle bisectors to find measures and distance relationships.
- Write equations for perpendicular bisectors.



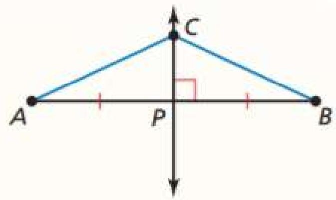


Theorems

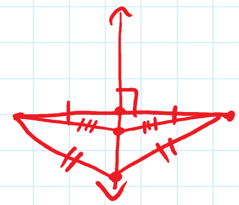
Theorem 6.1 Perpendicular Bisector Theorem

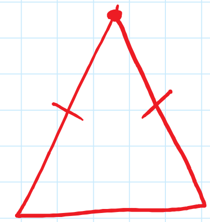
In a plane, if a point lies on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

If $\overleftrightarrow{CP}$ is the $\perp$ bisector of $\overline{AB}$, then $CA = CB$.



Proof p. 302

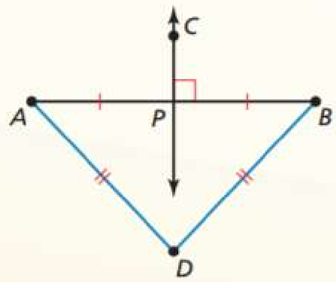


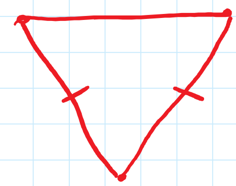


Theorem 6.2 Converse of the Perpendicular Bisector Theorem

In a plane, if a point is equidistant from the endpoints of a segment, then it lies on the perpendicular bisector of the segment.

If $DA = DB$, then point D lies on the $\perp$ bisector of $\overline{AB}$.

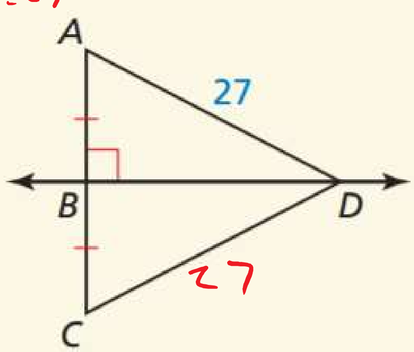




Proof Ex. 32, p. 308

Find each measure.

a. CD = 27



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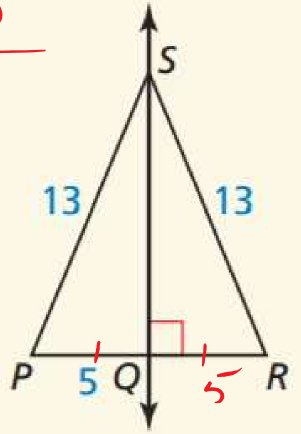
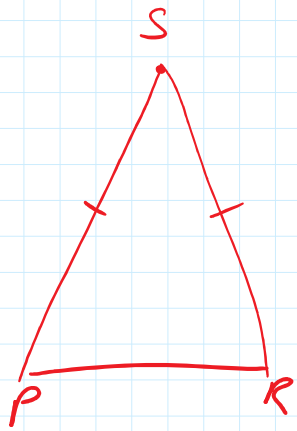
b. PR = 10

b. $PR = 10$





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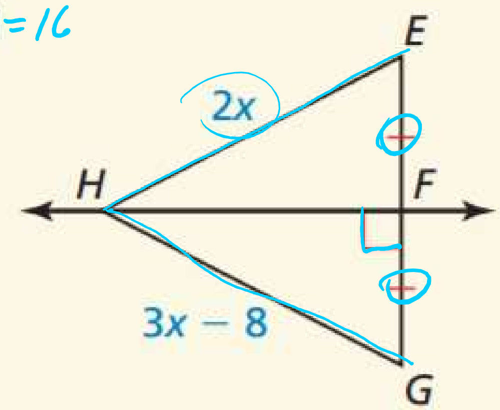
c. $GH = 16$



$2x = 3x - 8$

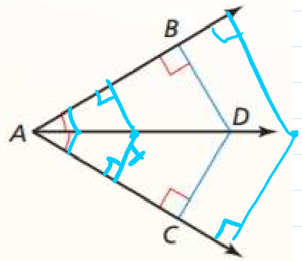
$x = 8$

Theorems

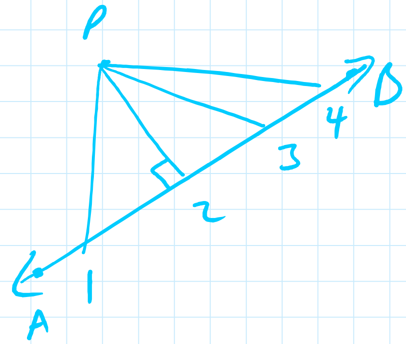
Theorem 6.3 Angle Bisector Theorem

If a point lies on the bisector of an angle, then it is equidistant from the two sides of the angle.

If $\overrightarrow{AD}$ bisects $\angle BAC$ and $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$, then $DB = DC$.



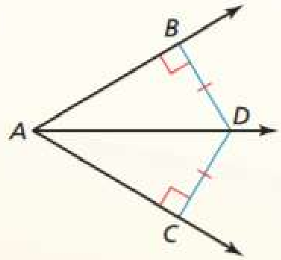
Proof Ex. 33(a), p. 308



Theorem 6.4 Converse of the Angle Bisector Theorem

If a point is in the interior of an angle and is equidistant from the two sides of the angle, then it lies on the bisector of the angle.

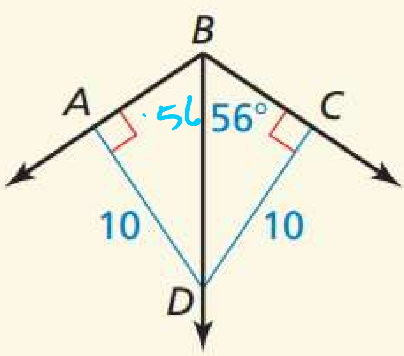
If $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$ and $DB = DC$, then $\overrightarrow{AD}$ bisects $\angle BAC$.

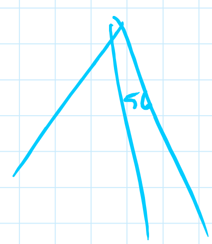


Proof Ex. 33(b), p. 308

Find each measure.

a. $m\angle ABC$ = 112°





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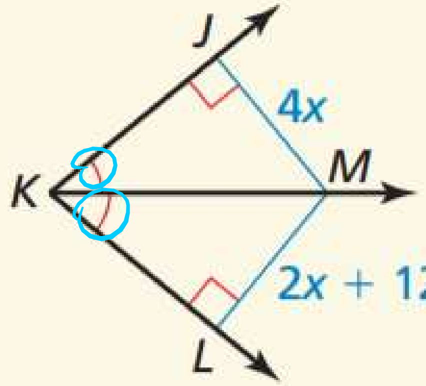
b. $JM = 24$



$2x + 12 = 4x$

$x = 6$

Write an equation of the perpendicular bisector of the segment with endpoints $D(5, -1)$ and $E(-11, 3)$.

$M_{(x,y)} = \left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$

$\left(\frac{-11+5}{2}, \frac{3+-1}{2}\right) = \left(\frac{-6}{2}, \frac{2}{2}\right)$

$(-3, 1)$

$m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2-y_1}{x_2-x_1} = \frac{3+1}{-11-5} = \frac{4}{-16} = -\frac{1}{4}$

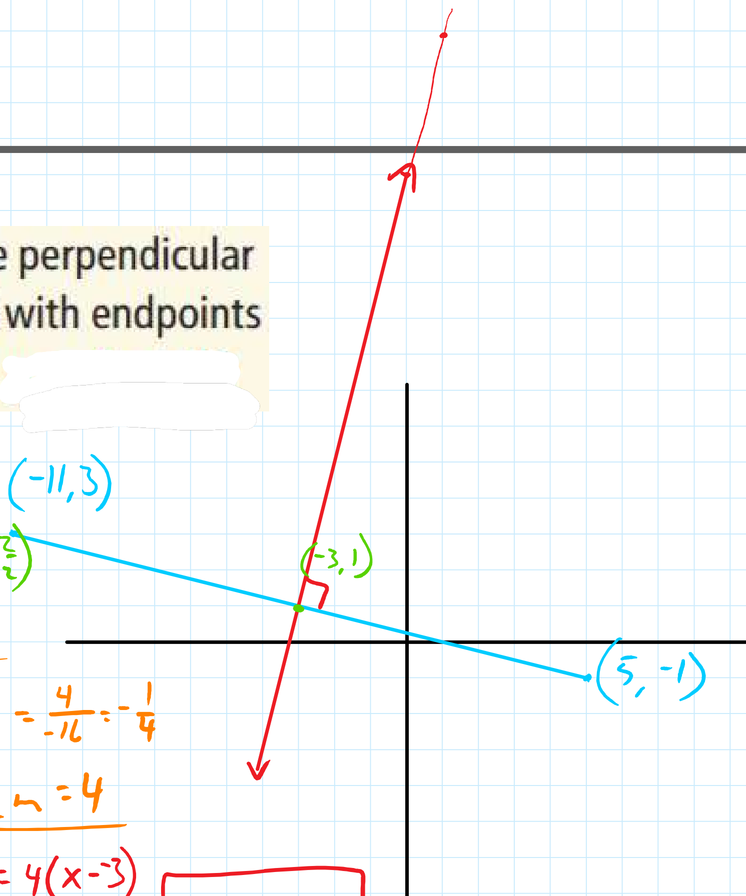
$\perp m = -\frac{1}{m} \qquad -\frac{1}{4} \rightarrow \frac{4}{1} = \perp m = 4$

$y - y_1 = m(x - x_1) \rightarrow y - 1 = 4(x - -3)$

$y - 1 = 4x + 12$

$+1 \qquad +1$

$\boxed{y = 4x + 13}$



Write an equation of the perpendicular bisector of the segment with endpoints $(-1, -5)$ and $(3, -1)$.

$M_{(x,y)} = \left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$

$\left(\frac{-1+3}{\ }\right) \left(\ \right) = (1, -3)$

$M_{(x,y)} \left(\frac{\quad}{2} , \frac{\quad}{2} \right)$

$\left(\frac{-1+3}{2} , \frac{-5+-1}{2} \right) = \left(\frac{2}{2} , \frac{-6}{2} \right) = (1,-3)$

$m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

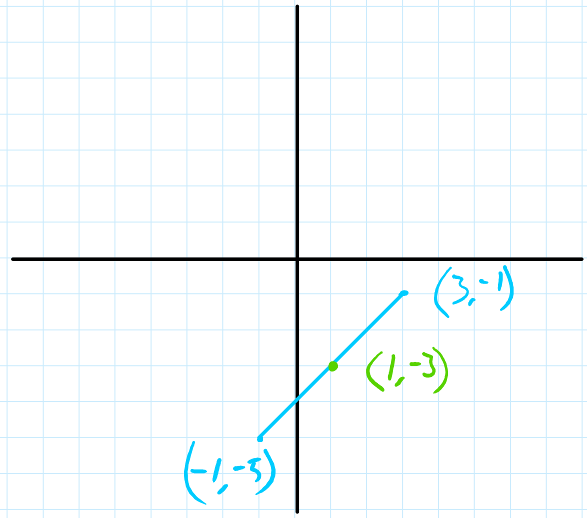
$\frac{-5 - -1}{-1-3} = \frac{-4}{-4} = 1$

$\perp m = -\frac{1}{m} \rightarrow -1$

$y - y_1 = m(x - x_1) \qquad y - {}^-3 = -1(x-1)$

$y = mx + b \qquad y + 3 = -x + 1$

$\boxed{y = -x - 2}$



Practice sec 6.1 **pg. 306: 1-20A**

What You Will Learn

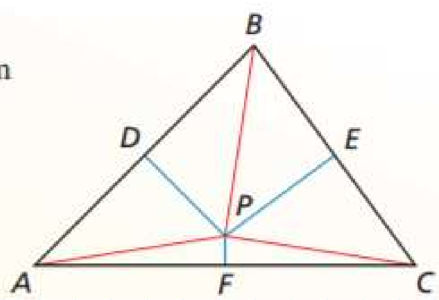
- Use and find the circumcenter of a triangle.
- Use and find the incenter of a triangle.

Theorem 6.5 Circumcenter Theorem

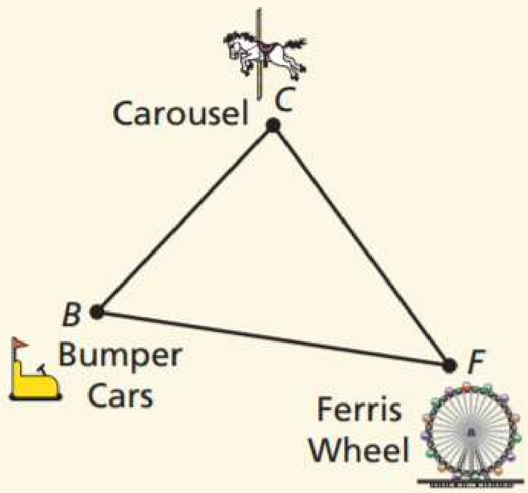
The circumcenter of a triangle is equidistant from the vertices of the triangle.

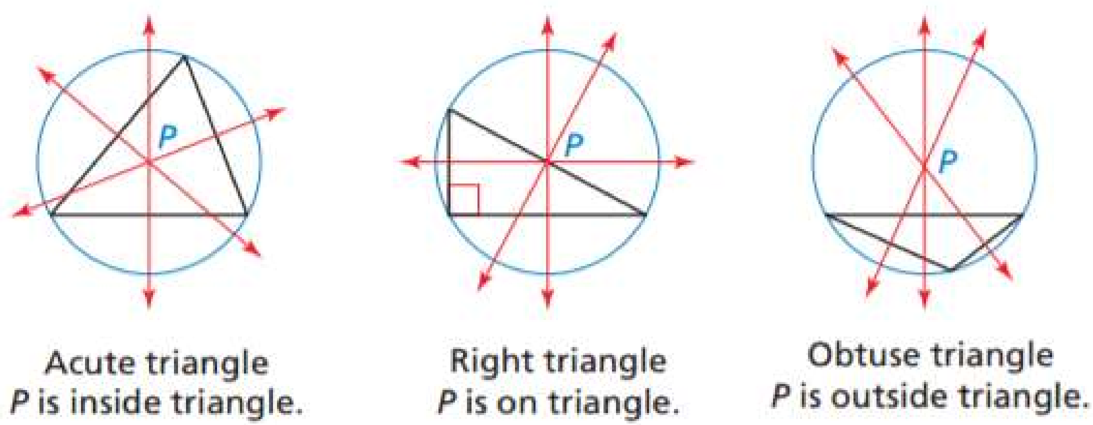
If $\overline{PD}$, $\overline{PE}$, and $\overline{PF}$ are perpendicular bisectors, then $PA = PB = PC$.

Proof p. 310



A carnival operator wants to locate a food stand so that it is the same distance from the carousel (C), the Ferris wheel (F), and the bumper cars (B). Find the location of the food stand (S).





Find the coordinates of the circumcenter of $\triangle DEF$ with vertices $D(6, 4)$, $E(-2, 4)$, and $F(-2, -2)$.

Find the coordinates of the circumcenter of the triangle with the given vertices.

2. $R(-2, 5)$, $S(-6, 5)$, $T(-2, -1)$

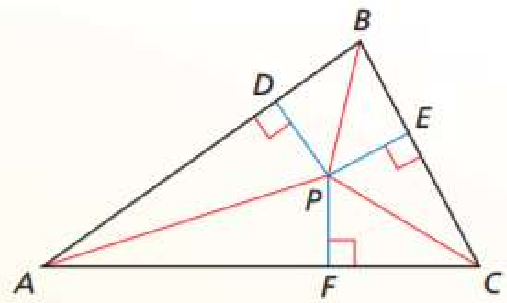
3. $W(-1, 4)$, $X(1, 4)$, $Y(1, -6)$

Theorem 6.6 Incenter Theorem

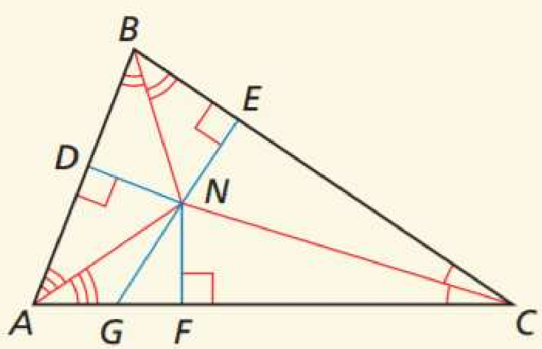
The incenter of a triangle is equidistant from the sides of the triangle.

If $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$ are angle bisectors of $\triangle ABC$, then $PD = PE = PF$.

Proof Ex. 38, p. 317



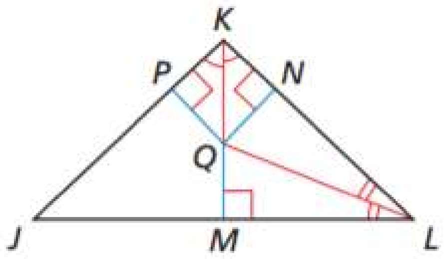
In the figure shown, $NE = 6x + 1$ and $NF = 4x + 15$.



a. Find *ND*.

b. Can $NB = 40$? Explain your reasoning.

In the figure shown, $QM = 3x + 8$ and $QN = 7x + 2$. Find QP.



Practice sec 6.2 **pg. 315: 1-7A, 9, 11-16A, 29-32A**