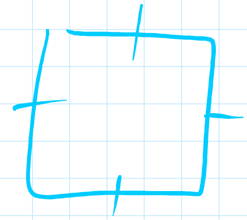
11.3 Areas of Polygons

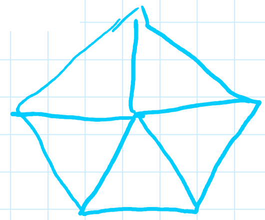
Tuesday, September 5, 2017 1:18 PM



What You Will Learn

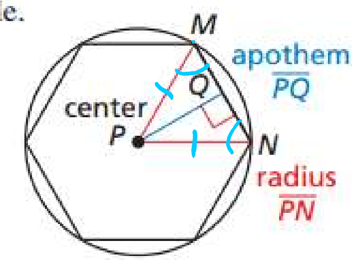
- Find angle measures in regular polygons.
- Find areas of regular polygons.

$A = \frac{bh}{2} = \frac{1}{2}bh$

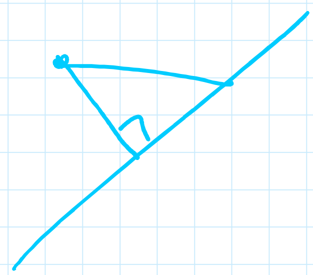


Finding Angle Measures in Regular Polygons

The diagram shows a regular polygon inscribed in a circle. The **center of a regular polygon** and the **radius of a regular polygon** are the center and the radius of its circumscribed circle.



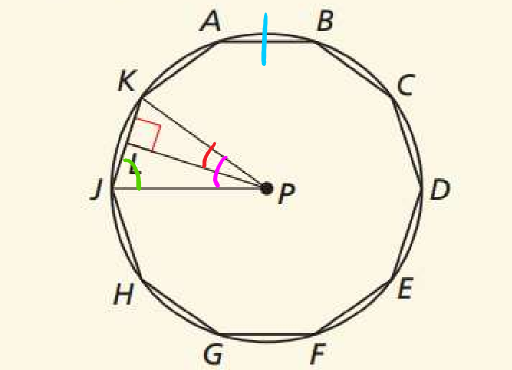
$\angle MPN$ is a central angle.

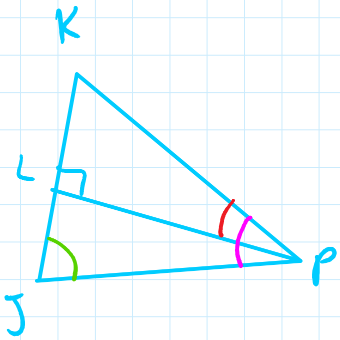


The distance from the center to any side of a regular polygon is called the **apothem of a regular polygon**. The apothem is the height to the base of an isosceles triangle that has two radii as legs. The word “apothem” refers to a segment as well as a length. For a given regular polygon, think of *an* apothem as a segment and *the* apothem as a length.

A **central angle of a regular polygon** is an angle formed by two radii drawn to consecutive vertices of the polygon. To find the measure of each central angle, divide 360° by the number of sides.

In the diagram, polygon *ABCDEFGHJK* is a regular decagon inscribed in $\odot P$. Find each angle measure.





$m\angle KPJ$: $\frac{360}{10}$ = 36°

$m\angle LPK$: $\frac{36}{2}$ = 18

$m\angle LJP$: 180 − 36 = 144, $\frac{144}{2}$ = 72

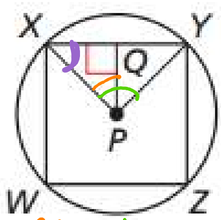
a. $m\angle KPJ = 36°$

b. $m\angle LPK = 18°$

c. $m\angle LJP = 72°$

$A = \frac{1}{2}bh$ SOH-CAH-TOA

In the diagram, *WXYZ* is a square inscribed in $\odot P$.



3. Identify the center, a radius, an apothem, and a central angle of the polygon.

4. Find $m\angle XPY$, $m\angle XPQ$, and $m\angle PXQ$.

center - P

radius - $\overline{XP}$ or $\overline{YP}$

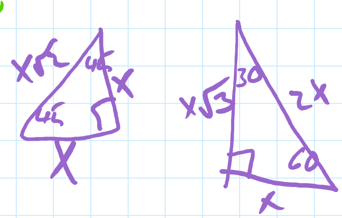
apothem - $\overline{PQ}$

central ∠ - ∠XPY

$m\angle XPY$: $\frac{360}{4}$ = 90°

$m\angle XPQ$: $\frac{90}{2}$ = 45°

$m\angle PXQ$: $90 + x + 45 = 180$, $x = 45$, 45°



Finding Areas of Regular Polygons

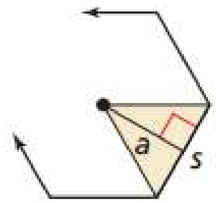
You can find the area of any regular n-gon by dividing it into congruent triangles.

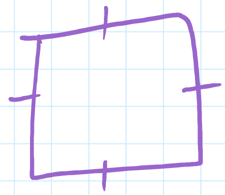
$A =$ Area of one triangle • Number of triangles

$= \left(\frac{1}{2} \cdot s \cdot a\right) \cdot n$ — Base of triangle is s and height of triangle is a. Number of triangles is n.

$= \frac{1}{2} \cdot a \cdot (n \cdot s)$ — Commutative and Associative Properties of Multiplication

$= \frac{1}{2}a \cdot P$ — There are n congruent sides of length s, so perimeter P is $n \cdot s$.



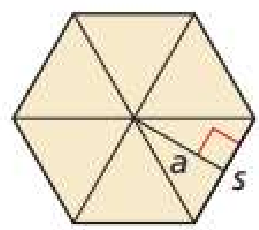


$4 \cdot 5 = 20\text{in}$

Area of a Regular Polygon

The area of a regular n-gon with side length s is one-half the product of the apothem a and the perimeter P.

$$A = \frac{1}{2}aP, \text{ or } A = \frac{1}{2}a \cdot ns$$



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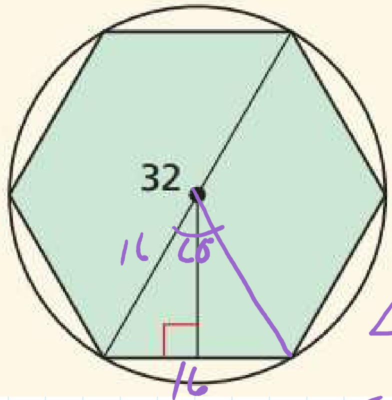
A regular hexagon is inscribed in a circle with a diameter of 32 units. Find the area of the hexagon.

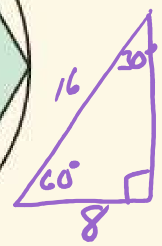


$A = \frac{1}{2} a P$

$\frac{1}{2} \; 8\sqrt{3} \; 96$

$4\sqrt{3} \; 96$

$\boxed{A = 384\sqrt{3}}$



$a = 8\sqrt{3}$

$P = 96$

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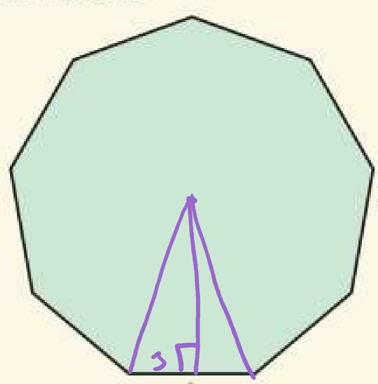
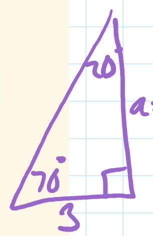
9 A mirror is in the shape of a regular nonagon with 6-inch sides. What is the area of the mirror?



~~SOH~~ - ~~CAH~~ - TOA

$\tan 20 = \frac{3}{a}$



$a(.3640) = \left(\frac{3}{\cancel{a}}\right)\cancel{a}$

$\frac{.3640a}{.3640} = \frac{3}{.3640}$

$a = 8.2$

$\frac{360}{9}$

40

$A = \frac{1}{2} a P$

$= \frac{1}{2} \; 8.2 \cdot 54$

$(4.1) \, 54$

$\boxed{A = 221.4 \text{ in}^2}$

Practice *sec 11.3* **pg. 614: 7-24A**