11.1.1 Circumference and Arc Length

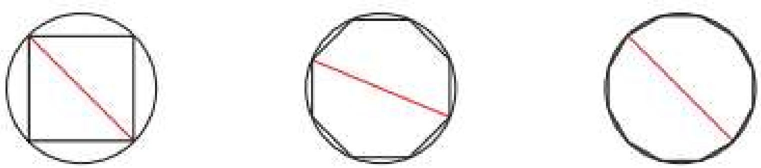
Monday, January 23, 2017 2:59 PM

What You Will Learn

- Use the formula for circumference. $2\pi r = C$
- Use arc lengths to find measures.
- Solve real-life problems.
- Measure angles in radians.

Using the Formula for Circumference

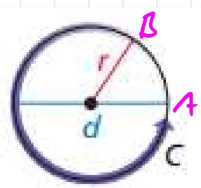
The **circumference** of a circle is the distance around the circle. Consider a regular polygon inscribed in a circle. As the number of sides increases, the polygon approximates the circle and the ratio of the perimeter of the polygon to the diameter of the circle approaches $\pi \approx 3.14159\ldots$.



For all circles, the ratio of the circumference C to the diameter d is the same. This ratio is $\frac{C}{d} = \pi$. Solving for C yields the formula for the circumference of a circle, $C = \pi d$. Because $d = 2r$, you can also write the formula as $C = \pi(2r) = 2\pi r$.

Circumference of a Circle

The circumference C of a circle is $C = \pi d$ or $C = 2\pi r$, where d is the diameter of the circle and r is the radius of the circle.



$C = \pi d = 2\pi r$

$m\,\overset{\frown}{BAB}$

round to nearest hundredth

Find each indicated measure.

a. circumference of a circle with a radius of 11 inches

$C = 2\pi r$

$= 2\pi 11$

$= 22\pi$

$= 69.1150$

69.12in

b. radius of a circle with a circumference of 4 millimeters

$C = 2\pi r$

$\frac{4mm}{2\pi} = \frac{\cancel{2\pi} r}{\cancel{2\pi}}$

~~$4 \div 2\pi$ $\boxed{Enter}$~~

~~6.28~~

$4 \div (2\pi)$ $\boxed{Enter}$

.64mm

~~6.28~~

.6366

.64mm

Using Arc Lengths to Find Measures

An **arc length** is a portion of the circumference of a circle. You can use the measure of the arc (in degrees) to find its length (in linear units).

$\widehat{AB}$ vs. $m\,\widehat{AB}$ vs. $l\widehat{AB}$ $\ell\widehat{AB}$

degrees

arc length $\widehat{AB}$

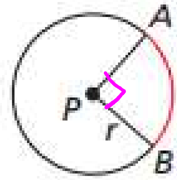
units: length

i.e. inches feet mm

miles

Arc Length

In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.



$\ell \widehat{AB}$ $C = 100$ in

$\frac{\text{part}}{\text{whole}}$ $\frac{\text{Arc length of } \widehat{AB}}{2\pi r \; C} = \frac{m\widehat{AB}}{360°}$, or $\frac{\text{part}}{\text{whole}}$ $\ell \widehat{AB} = 25$ in

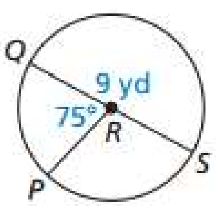
$m\widehat{AB} = 90°$

$$\text{Arc length of } \widehat{AB} = \frac{m\widehat{AB}}{360°} \cdot 2\pi r$$

$\frac{90°}{360°} = \frac{1}{4} = 25\%$

Find the indicated measure.

3. arc length of $\widehat{PQ}$ **4.** circumference of $\odot N$ **5.** radius of $\odot G$



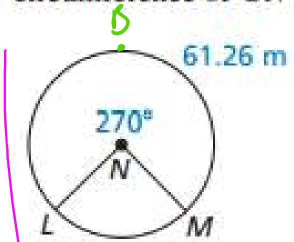
$C = 2\pi r$
$C = d\pi$
$C = 9\pi$
$C = 28.27$

$\frac{\ell \widehat{PQ}}{C} = \frac{m \widehat{PQ}}{360°}$

$\frac{\ell \widehat{PQ}}{28.27 \text{ yd}} = \frac{75°}{360°}$

$\frac{\ell \widehat{PQ}(360°)}{360°} = \frac{(28.27 \text{ yd})75°}{360°}$

$\boxed{\ell \widehat{PQ} \approx 5.89 \text{ yd}}$



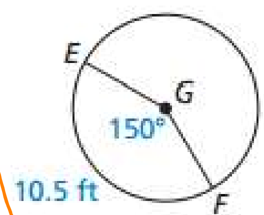
$\frac{\ell \widehat{LBM}}{C} = \frac{m \widehat{LBM}}{360°}$

$\frac{61.26 \text{ m}}{C} = \frac{\cancel{270°}^{3}}{\cancel{360°}_{4}}$

$3C = (61.26 \text{ m})4$

$\frac{3C}{3} = \frac{245.04 \text{ m}}{3}$

$\boxed{C = 81.68 \text{ m}}$



$\frac{\ell \widehat{EF}}{C} = \frac{m \widehat{EF}}{360°}$

$\frac{10.5 \text{ ft}}{C} = \frac{\cancel{150°}^{5}}{\cancel{360°}_{12}}$

$5C = (10.5 \text{ ft})12$

$\frac{\cancel{5}C}{\cancel{5}} = \frac{126 \text{ ft}}{5}$

$C = 25.2$ ft

$C = 2\pi r$

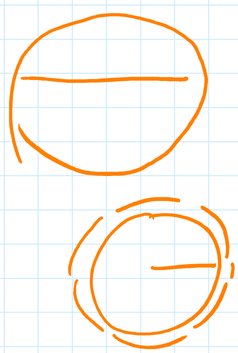
$\frac{25.2 \text{ ft}}{2\pi} = \frac{\cancel{2\pi} r}{\cancel{2\pi}}$

$\boxed{4.01 \text{ ft} = r}$

What is a *Radian*?

measurement of angle radius

$\frac{c}{d} = \pi$



Converting between Degrees and Radians

Degrees to radians

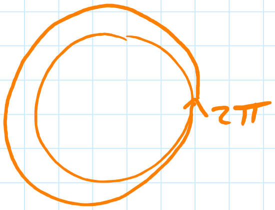
Multiply degree measure by

$$\frac{2\pi \text{ radians}}{360^\circ}, \text{ or } \frac{\pi \text{ radians}}{180^\circ}.$$

Radians to degrees

Multiply radian measure by

$$\frac{360^\circ}{2\pi \text{ radians}}, \text{ or } \frac{180^\circ}{\pi \text{ radians}}.$$



$\frac{5}{5} = 1$

$\frac{1}{1} = 1 \quad \frac{x}{x} = 1 \quad \frac{♡}{♡} = 1$

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8. Convert 15° to radians.

$15° \cdot 1$

$\frac{15°}{1} \cdot \frac{\pi}{180°} = \frac{\cancel{15}^{1}\pi}{\cancel{180}_{12}}$

$\boxed{\frac{\pi}{12}}$

9. Convert $\frac{4\pi}{3}$ radians to degrees.

$\frac{4\pi}{3} \cdot 1$

$\frac{4\pi}{3} \cdot \frac{180°}{\pi} = \frac{4\cancel{\pi}\,\cancel{180}^{60}°}{\cancel{3}\cancel{\pi}}$

$\boxed{240°}$

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a. Convert 30° to radians.

$\frac{180°}{\pi}$

$\frac{\pi}{180°}$

$\frac{\cancel{30}^{1}°}{1} \cdot \frac{\pi}{\cancel{180}_{6}}$

$\boxed{\frac{\pi}{6}}$

~~$\frac{1}{6}\pi$~~

b. Convert $\frac{3\pi}{8}$ radians to degrees.

$\frac{3\cancel{\pi}}{8} \cdot \frac{180°}{\cancel{\pi}}$

$\frac{540°}{8} = \boxed{67.5°}$

Practice *sec 11.1* **pg. 598:**
1-3A, 5-7EO, 8-11A,
19-22A