10.5 Angle Relationships in circles

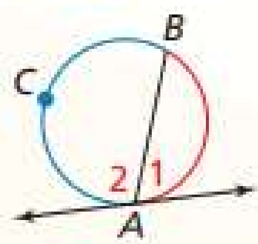
Tuesday, September 5, 2017 1:18 PM

What You Will Learn

- Find angle and arc measures.
- Use circumscribed angles.

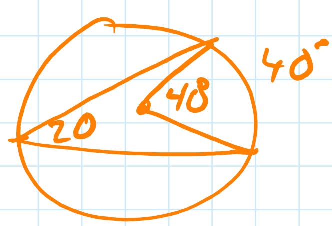
Theorem 10.14 Tangent and Intersected Chord Theorem

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one-half the measure of its intercepted arc.



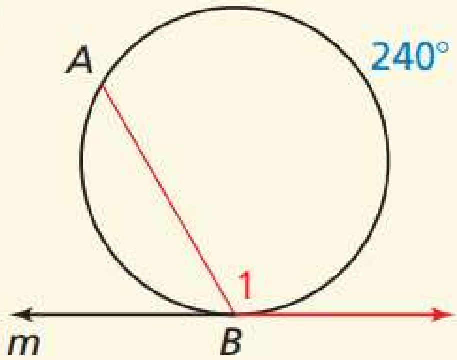
Proof Ex. 33, p. 568

$m\angle 1 = \frac{1}{2}m\widehat{AB}$ $m\angle 2 = \frac{1}{2}m\widehat{BCA}$



Line m is tangent to the circle. Find the measure of the red angle or arc.

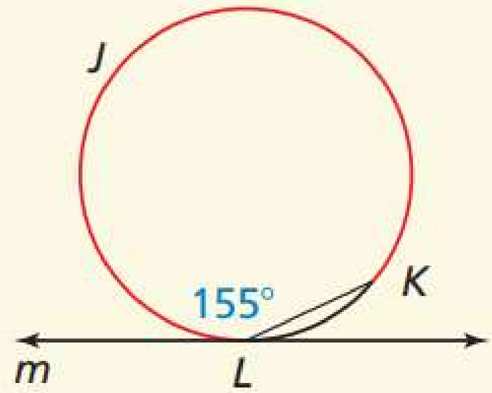
a.



$m\angle 1 = \frac{240}{2}$

$m\angle 1 = 120°$

b.



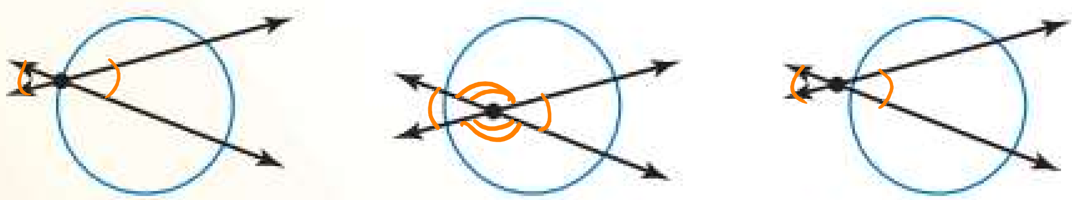
$\frac{m\overset{\frown}{LJK}}{2} = m\angle L$

$2\left(\frac{m\overset{\frown}{LJK}}{2}\right) = (155)2$

$m\overset{\frown}{LJK} = 310°$

Intersecting Lines and Circles

If two nonparallel lines intersect a circle, there are three places where the lines can intersect.

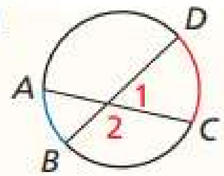


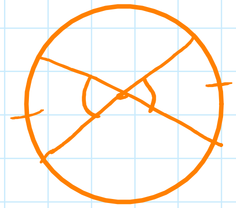
on the circle — inside the circle — outside the circle

$\underbrace{\text{on the circle}}_{\text{yesterday}}$ $\underbrace{\text{inside the circle} \quad \text{outside the circle}}_{\text{today}}$

Theorem 10.15 Angles Inside the Circle Theorem

If two chords intersect *inside* a circle, then the measure of each angle is one-half the *sum* of the measures of the arcs intercepted by the angle and its vertical angle.





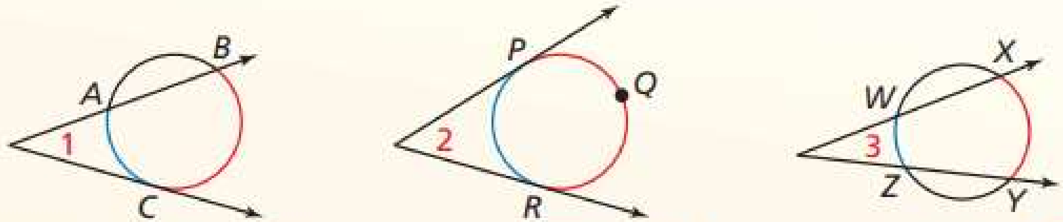
$m\angle 1 = \frac{1}{2}(m\widehat{DC} + m\widehat{AB})$,

$m\angle 2 = \frac{1}{2}(m\widehat{AD} + m\widehat{BC})$

Proof Ex. 35, p. 568

Theorem 10.16 Angles Outside the Circle Theorem

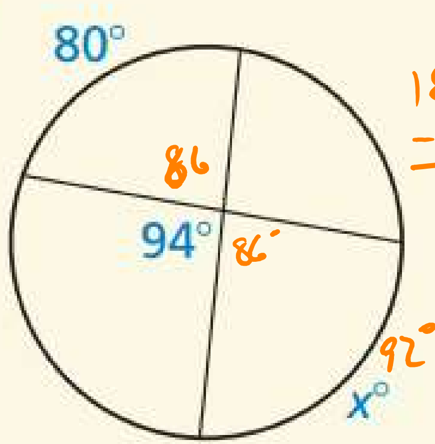
If a tangent and a secant, two tangents, or two secants intersect *outside* a circle, then the measure of the angle formed is one-half the *difference* of the measures of the intercepted arcs.



$m\angle 1 = \frac{1}{2}(m\widehat{BC} - m\widehat{AC})$ $m\angle 2 = \frac{1}{2}(m\widehat{PQR} - m\widehat{PR})$ $m\angle 3 = \frac{1}{2}(m\widehat{XY} - m\widehat{WZ})$

Find the value of *x*.

a.



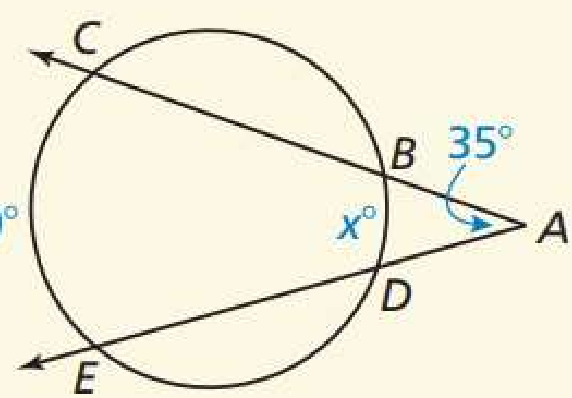
$$\begin{array}{r} 180 \\ -94 \\ \hline 86 \end{array}$$

$$2\left(\frac{x+80}{2}\right) = (86)2$$

$$x + 80 = 172$$

$$-80 \quad -80$$

b.



$$2\left(\frac{100-x}{2}\right) = (35°)2$$

$$\cancel{100} - x = 70$$

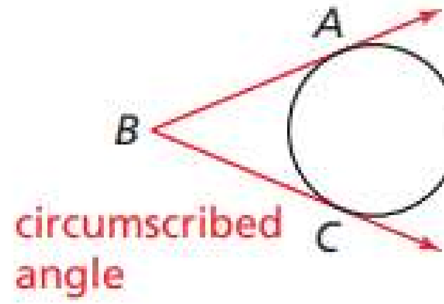
$$-\cancel{100} \quad -100$$

$-80 \quad -80$
$x = 92$

$(-1)(-x) = (-30)(-1)$
$x = 30$

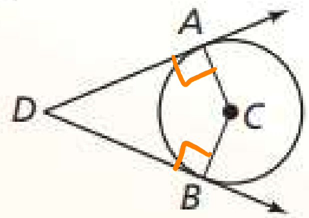
Circumscribed Angle

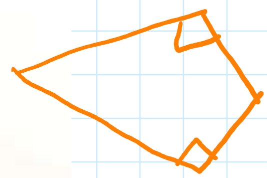
A **circumscribed angle** is an angle whose sides are tangent to a circle.



Theorem 10.17 Circumscribed Angle Theorem

The measure of a circumscribed angle is equal to 180° minus the measure of the central angle that intercepts the same arc.



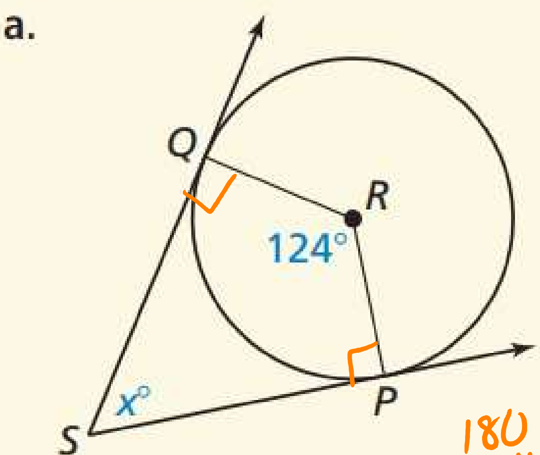


Proof Ex. 38, p. 568

$m\angle ADB = 180° - m\angle ACB$

Find the value of x.

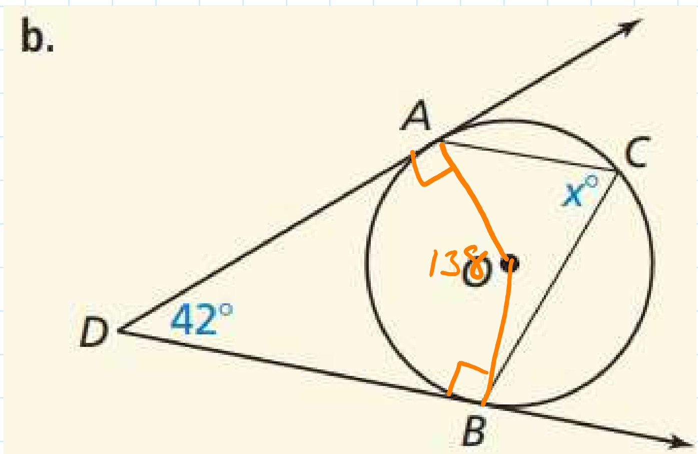
a.



180
-124
56

$x = 56$

b.



$42 + 90 + x + 90 = 360$
$222 + x = 360$

$x = 69$

$-222 \quad -222$

$x=138$

Practice sec 10.5 **pg. 566: 3-21EO**