What You Will Learn

- Find segment lengths using midpoints and segment bisectors.
- Use the Midpoint Formula.
- Use the Distance Formula.

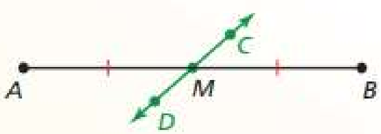
Midpoints and Segment Bisectors

The **midpoint** of a segment is the point that divides the segment into two congruent segments.

$\cong$

M is the midpoint of $\overline{AB}$.
So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.

A **segment bisector** is a point, ray, line, line segment, or plane that intersects the segment at its midpoint. A midpoint or a segment bisector *bisects* a segment.

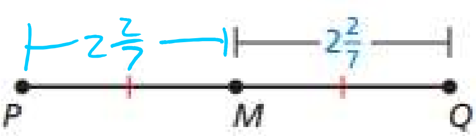


$\overline{DC}$ $\overline{CD}$ $\overleftrightarrow{DC}$ $\overleftrightarrow{CD}$ $\overrightarrow{MC}$ $\overrightarrow{DC}$

m $\overline{CD}$ $\overline{DC}$ m

$\overleftrightarrow{CD}$ is a segment bisector of $\overline{AB}$.
So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.

In the figure, $PM = 1.8$ mm. Identify the segment bisector of $\overline{PQ}$. Then find PQ.



$\boxed{PQ = 4\frac{4}{7}}$

Bisector → $\overrightarrow{MT}$
$\overline{MT}$
m

Bisector → $\overrightarrow{MT}$
$\overline{MT}$
M
$\overline{TM}$

$\boxed{PQ = 3.6\text{mm}}$

$\boxed{PQ = 4\frac{4}{7}}$

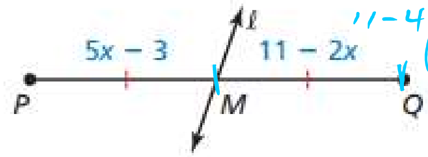
Point M is the midpoint of $\overline{AB}$. Find the length of $\overline{AB}$.

$3x-4 = \cancel{2x}+1$
$-2x \quad \cancel{-2x}$
$x\cancel{-4} = 1$
$\cancel{+4} \quad +4$
$x = 5$

$2x+1 \;;\; x=5$
$2\cdot5+1$
$10+1$
11

$\boxed{AB = 22}$

3. Identify the segment bisector of $\overline{PQ}$. Then find MQ.



$11-2x \;;\; x=2$
$11-2\cdot2$
$11-4$
$\boxed{7}$

Seg. bisector → ℓ, m

$5x-3 = 11\cancel{-2x}$
$+2x \quad \cancel{+2x}$
$7x\cancel{-3} = 11$
$\cancel{+3} \quad +3$
$\frac{\cancel{7}x}{\cancel{7}} = \frac{14}{7}$
$x = 2$

Using the Midpoint Formula

You can use the coordinates of the endpoints of a segment to find the coordinates of the midpoint.

$3, 5 \quad \frac{3+5}{2} = \frac{8}{2} = 4$

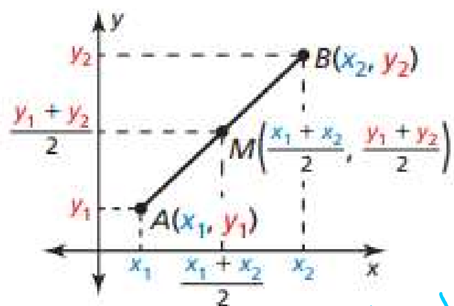
$3, 5, 7$

The Midpoint Formula

The coordinates of the midpoint of a segment are the averages of the x-coordinates and of the y-coordinates of the endpoints.

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint M of $\overline{AB}$ has coordinates

$$M_{(x,y)} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$



$A(1, 2) \quad B(5, 10)$
$m(3, 6)$

$A(7,2)$ $m(3,C)$

a. The endpoints of $\overline{AB}$ are $A(-8, 7)$ and $B(5, 1)$. Find the coordinates of the midpoint M.

$$m = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$x_m = \frac{-8+5}{2} \qquad y_m = \frac{7+1}{2}$$

$$x_m = \frac{-3}{2} \qquad y_m = 4$$

$$m = \left(\frac{-3}{2}, 4\right)$$

b. The midpoint of $\overline{PQ}$ is $M(2, -3)$. One endpoint is $P(4, 1)$. Find the coordinates of endpoint Q.

$$m = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$2(2) = \left(\frac{4+x_2}{\cancel{2}}\right)\cancel{2} \qquad 2(-3) = \left(\frac{1+y_2}{\cancel{2}}\right)\cancel{2}$$

$$\cancel{4} = \cancel{4} + x_2 \qquad -6 = \cancel{1} + y_2$$

$$\cancel{-4} \quad \cancel{-4} \qquad -1 \quad \cancel{-1}$$

$$0 = x_2 \qquad -7 = y_2$$

$$\boxed{Q = (0, -7)}$$

6. The endpoints of $\overline{CD}$ are $C(-4, 3)$ and $D(-6, 5)$. Find the coordinates of the midpoint M.

$$m_{(x,y)} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \quad (-5, 4) \qquad y_m = \frac{3+5}{2} = 4$$

$$x_m = \frac{-4+-6}{2} = -5$$

$$\boxed{m = (-5, 4)}$$

7. The midpoint of $\overline{TU}$ is $M(2, 4)$. One endpoint is $T(1, 1)$. Find the coordinates of endpoint U.

$$m_{(x,y)} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$4 = \frac{1+y_2}{2} \qquad 2 = \frac{1+x_2}{2}$$

$$8 = 1+y \qquad 4 = 1+x_2$$

$$-1 \quad -1 \qquad -1 \quad -1$$

$$7 = y \qquad 3 = x_2$$

$$\boxed{U = (3, 7)}$$

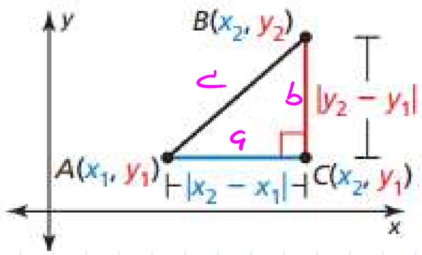
Using the Distance Formula

You can use the Distance Formula to find the distance between two points in a coordinate plane.

The Distance Formula

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the distance between A and B is

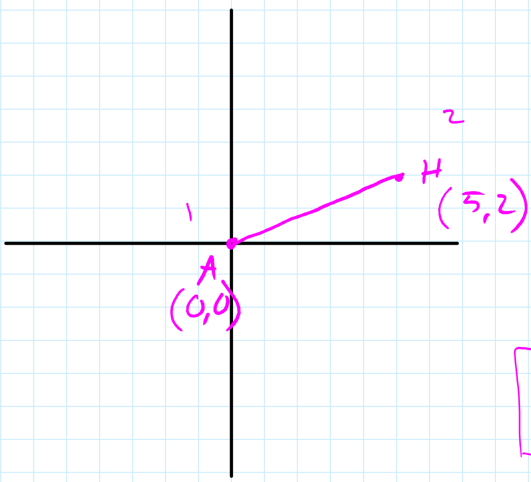
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



$$c^2 = \sqrt{a^2 + b^2}$$

$$\boxed{c^2 = a^2 + b^2}$$

You bicycle 5 miles east and then 2 miles north from your apartment to a friend's house. Estimate the distance between your friend's house and your ~~school~~ Apartment.



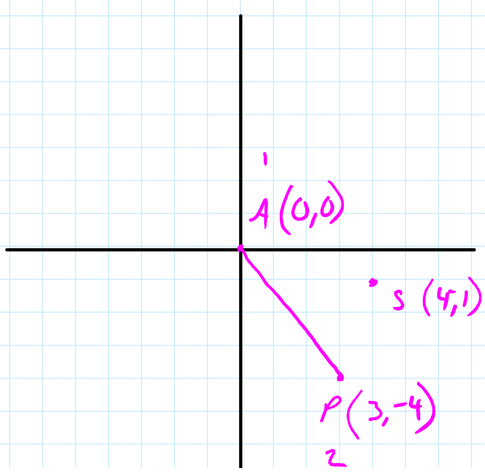
$$AH = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(5-0)^2 + (2-0)^2}$$

$$\sqrt{25 + 4}$$

$$\boxed{AH = \sqrt{29}}$$

Your school is 4 miles east and 1 mile south of your apartment. A park is 3 miles east and 4 miles south of your apartment. Find the distance between the park and your apartment.



$$AP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3-0)^2 + (-4-0)^2}$$

$$\sqrt{(3)^2 + (-4)^2}$$

$$\sqrt{9 + 16}$$

$$\sqrt{25}$$

$P(3,-4)$
2

$\sqrt{25}$

5

$\boxed{AP = 5}$

Practice sec 1.3

pg. 24: 1-9EO,
15-29EO, 46-49A